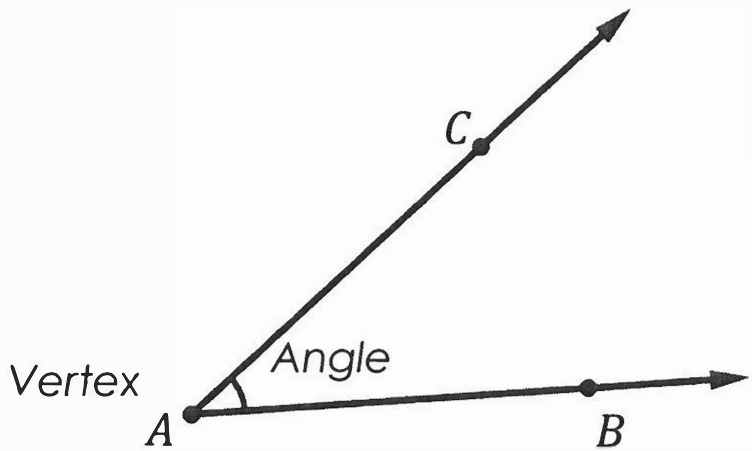
Section 2 – Topic 1
Introduction to Angles – Part 1

Consider the figure of angle A below.



What observations can you make about angle A?

Vertex is A. It consists of $\overrightarrow{AC}$ and $\overrightarrow{AB}$

How else do you think we can name angle A?

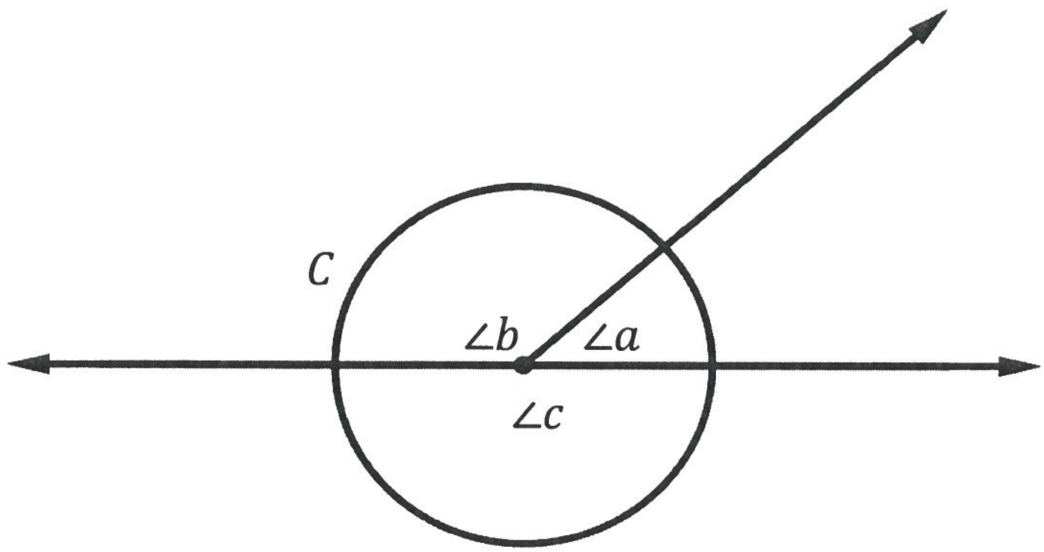
$\angle CAB$ or $\angle BAC$

Why do you think we draw an arc to show angle A?

Arc is a segment of a circle.

Like circles, angles are measured in degrees since they measure the amount of rotation around the center.

Consider the figure below.



Use the figure to answer the following questions.

What is the measure of circle C? 360°

What is the measure of $\angle a + \angle b + \angle c$? 360°

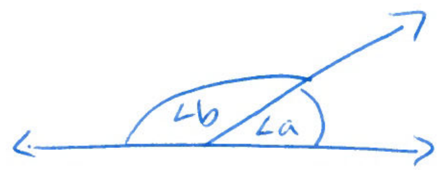
How many degrees is half of a circle? 180°

What is the measure of $\angle a + \angle b$? 180°

Two positive angles that form a straight line together are called supplementary angles.

➢ When added together, the measures of these angles is 180 degrees, forming a linear pair.

Draw an example of ***supplementary angles*** that form a ***linear pair***.



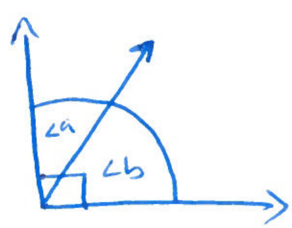
A quarter-circle is a right angle.

How many degrees are in a right angle?

90°

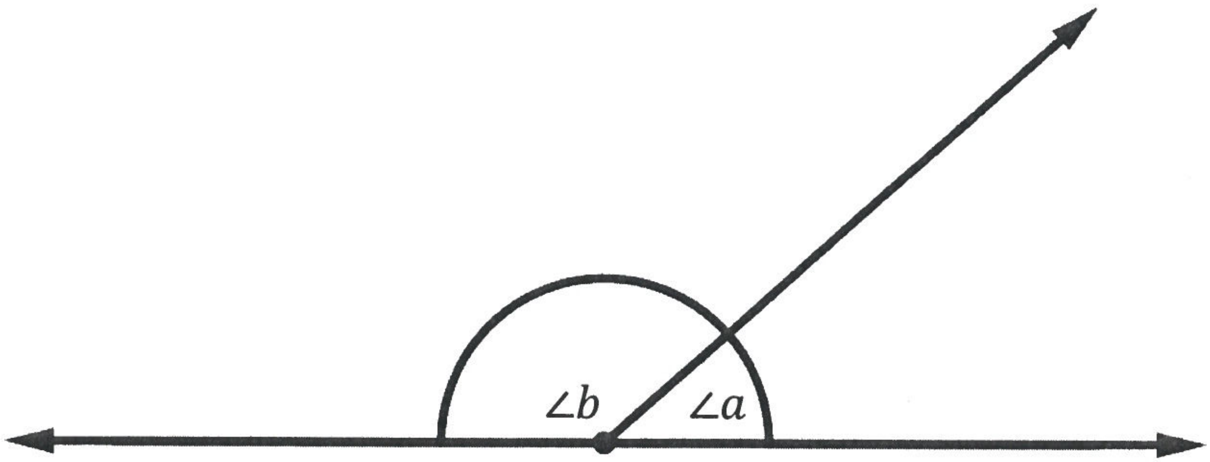
Two positive angles that together form a right angle are called Complimentary angles.

Draw an example of ***complementary angles***.



Let's Practice!

1. In the figure below, $m\angle a = 7x + 5$ and $m\angle b = 28x$. The angles are supplementary.



Find the value of x and the measure of $\angle a$ and $\angle b$ in degrees.

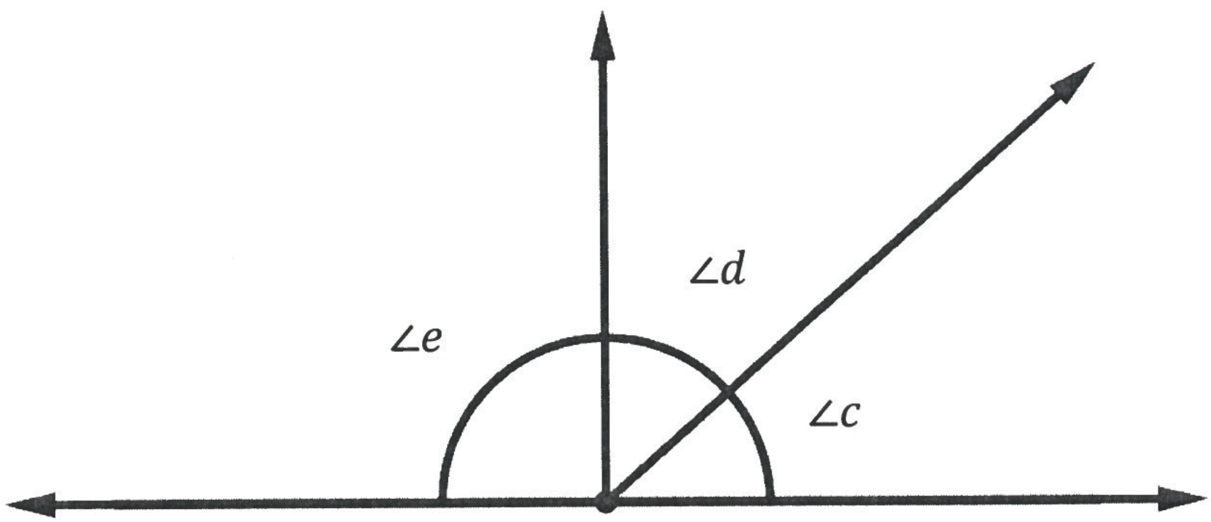
$$7x + 5 + 28x = 180$$
$$35x + 5 = 180$$
$$-5 \quad\quad -5$$
$$\frac{35x}{35} = \frac{175}{35}$$
$$x = 5$$

$$m\angle a = 7(5) + 5 = 40°$$
$$m\angle b = 28(5) = 140°$$

STUDY EDGE TIP

When we refer to the angle as $\angle ABC$, we mean the actual angle object. If we want to talk about the size or the measure of the angle in degrees, we often write it as $m\angle ABC$.

2. In the figure below, $m\angle c = 9x - 3$ and $m\angle d = 8x + 9$.



a. If $x = 5$, are $\angle c$ and $\angle d$ complementary? Justify your answer.

$m\angle c = 9(5) - 3 = 42°$

$m\angle d = 8(5) + 9 = 49°$

$42° + 49° = 91°$

Not complementary!

$91° \neq 90°$

b. If $\angle c$, $\angle d$, and $\angle e$ form half a circle, then what is the measure of $\angle e$ in degrees? 180°

$42° + 49° + m\angle e = 180°$

$91° + m\angle e = 180°$

$-91° \qquad -91°$

$m\angle e = 89°$

Try It!

3. Angle A is 20 degrees larger than angle B. If A and B are complementary, what is the measure of angle A?

$A = 20 + B$

$A + B = 90°$

$(20 + B) + B = 90°$

$20 + 2B = 90°$

$-20 \qquad -20$

$2B = 70$

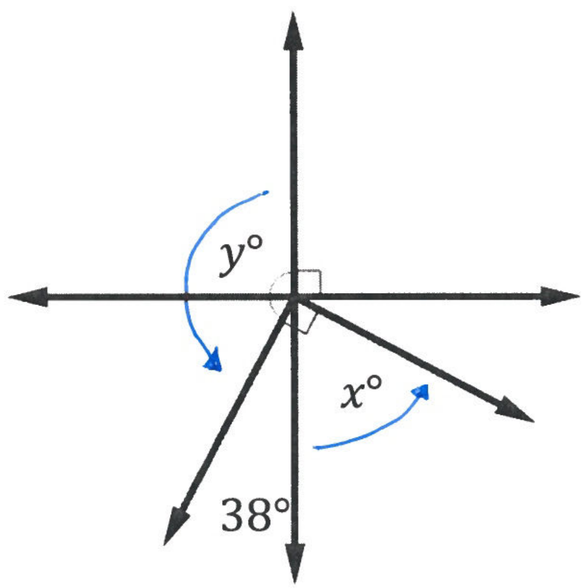
$\frac{2B}{2} = \frac{70}{2}$

$B = 35°$

$A = 20 + 35 = 55$

$\boxed{m\angle A = 55°}$

4. Consider the figure below.



If y stretches from the positive y-axis to the ray that makes the 38° angle, set up and solve an appropriate equation for x and y.

$y + 38 = 180$

$-38 \qquad -38$

$\boxed{m\angle y = 142°}$

$x + 38 = 90°$

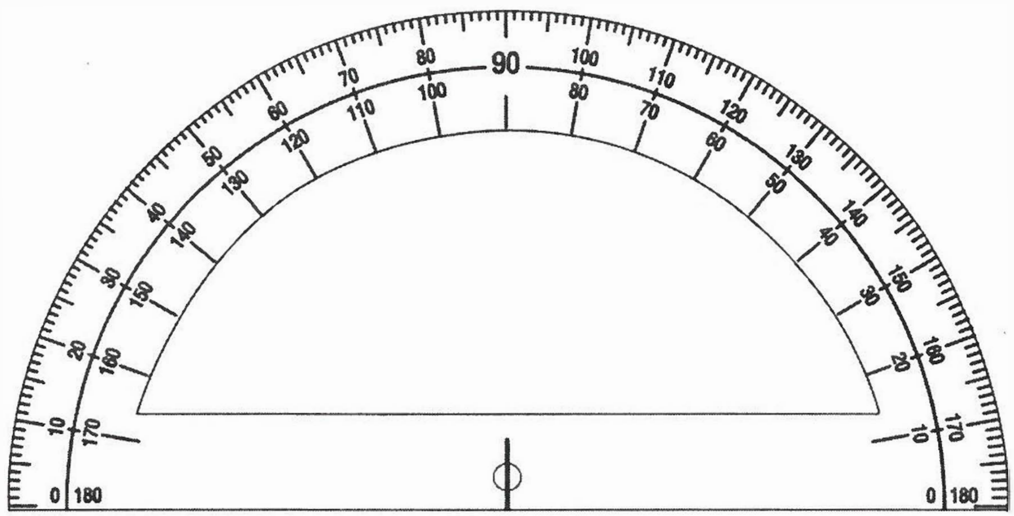
$-38 \qquad -38$

$m\angle x = 52°$

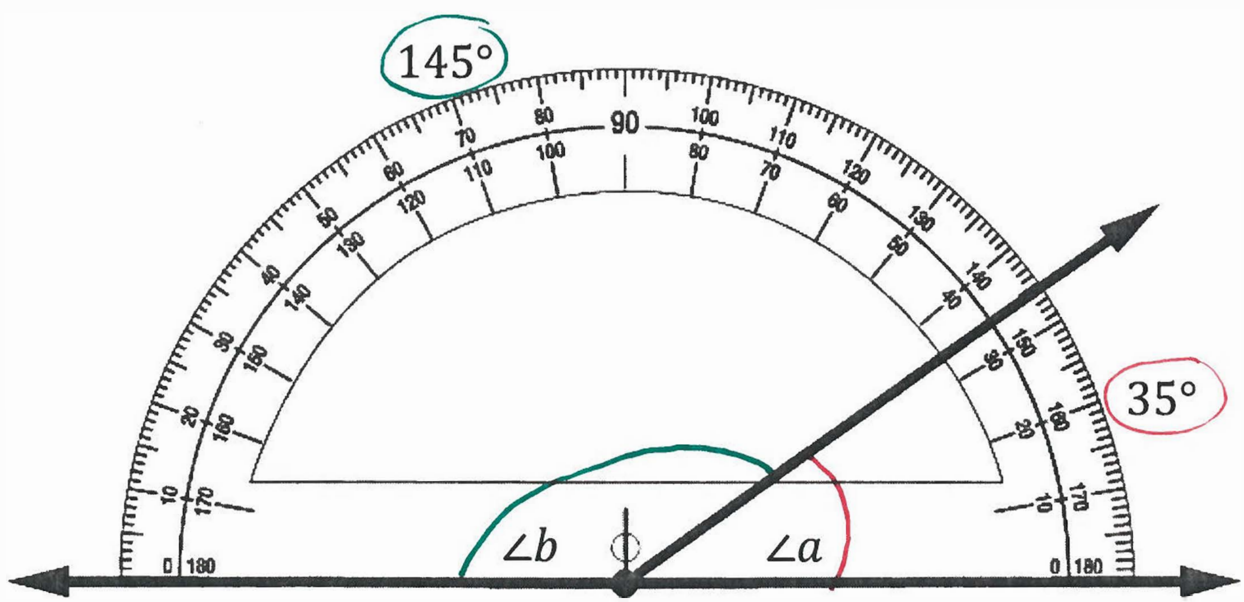
Section 2 – Topic 2
Introduction to Angles – Part 2

Measuring and classifying angles

- We often use a protractor to measure angles.



To measure an angle, we line up the central mark on the base of the ***protractor*** with the vertex of the angle we want to measure.

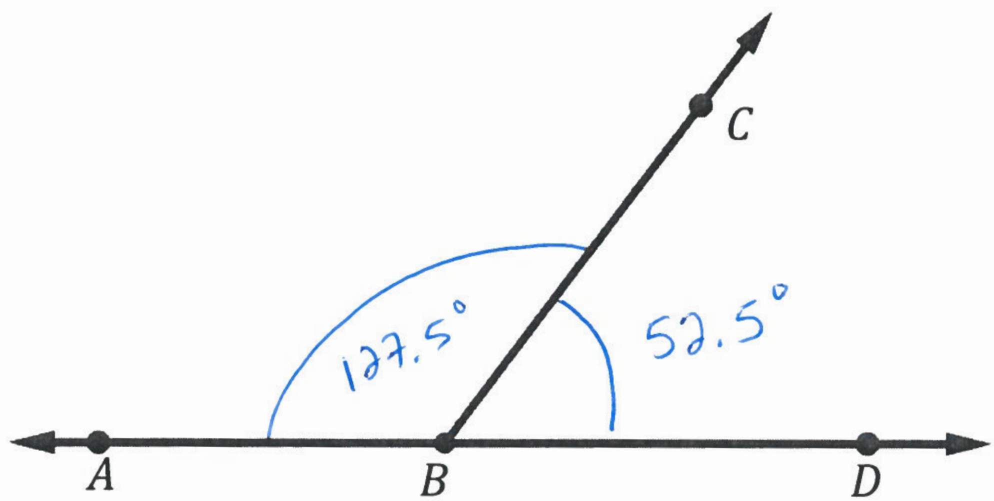


TAKE NOTE! *Postulates & Theorems*

The Protractor Postulate

The measure of the angle is the absolute value of the difference of the real numbers paired with the sides of the angle, because the parts of angles formed by rays between the sides of a linear pair add to the whole, 180°.

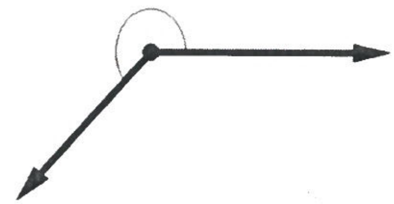
Label, write and measure the angles in the following figure.



Match each of the following words to the most appropriate figure represented below. Write your answer in the space provided below each figure.

Acute	**Obtuse**	**Right**	**Straight**	**Reflex**

Obtuse



Reflex

Acute

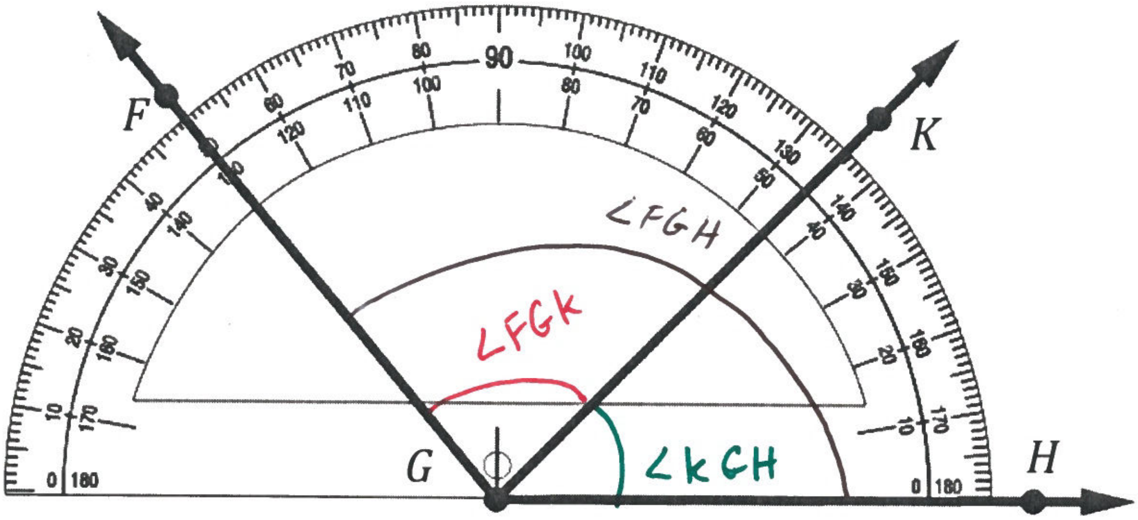
Right

Straight

- An angle that measures less than 90° is acute.
- An angle that measures greater than 90° but less than 180° is obtuse.
- An angle that measures exactly 90° is right.
- An angle of exactly 180° is straight.
- An angle greater than 180° is called a reflex angle.

Let's Practice!

1. Use the figure below to fill in the blanks that define angles $\angle FGK$, $\angle FGH$, and $\angle KGH$ as acute, obtuse, right or straight.



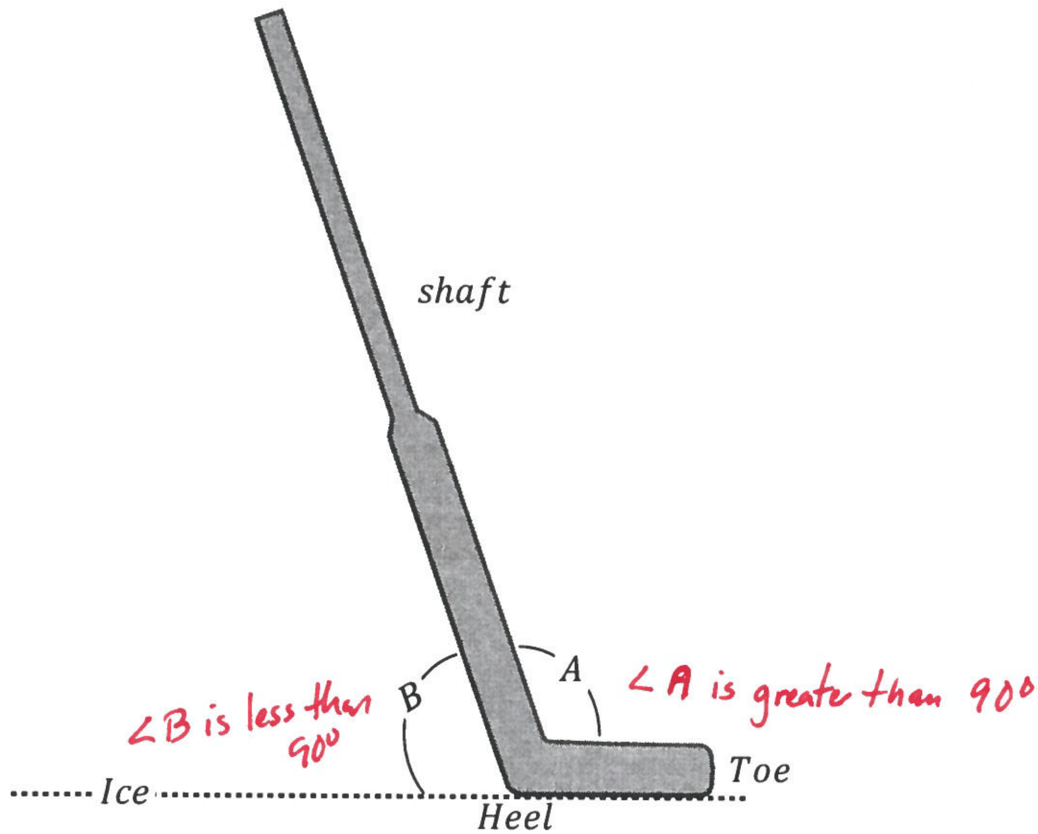
a. $\angle FGK$ is a(n) ___acute___ angle.

b. $\angle FGH$ is a(n) ___obtuse___ angle.

c. $\angle KGH$ is a(n) ___acute___ angle.

Try It!

2. A hockey stick comes into contact with the ice in such a way that the shaft makes an angle with the ice, labeled as angle B in the figure below. The angle between the shaft and the toe of the hockey stick Is labeled as A.



a. Determine the type of angle that is between the ice and the shaft. Is it acute, right, obtuse, or straight?

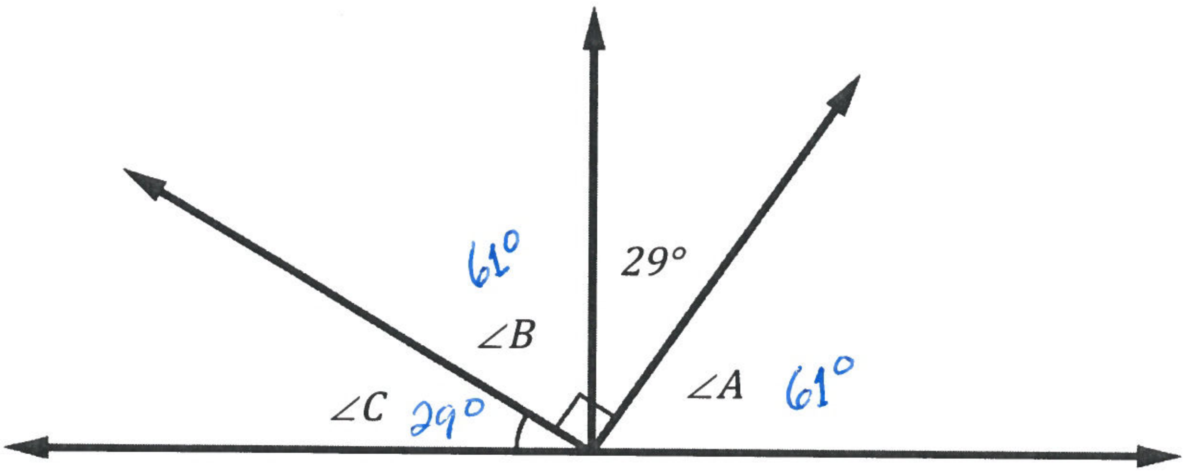
Acute

b. Determine the type of angle that is between the shaft and the toe. Is it acute, right, obtuse, or straight?

Obtuse

BEAT THE TEST!

1. Consider the figure below.



If $\angle B$ and $\angle C$ are complementary, then:

The measure of $\angle A$ is $\boxed{61^\circ}$.

The sum of $m\angle A$ and $m\angle B$ is $\boxed{122^\circ}$.

The sum of $\underset{61^\circ}{m\angle A}$, $\underbrace{m\angle B\text{, and } m\angle C}_{90^\circ}$ is $\boxed{151^\circ}$.

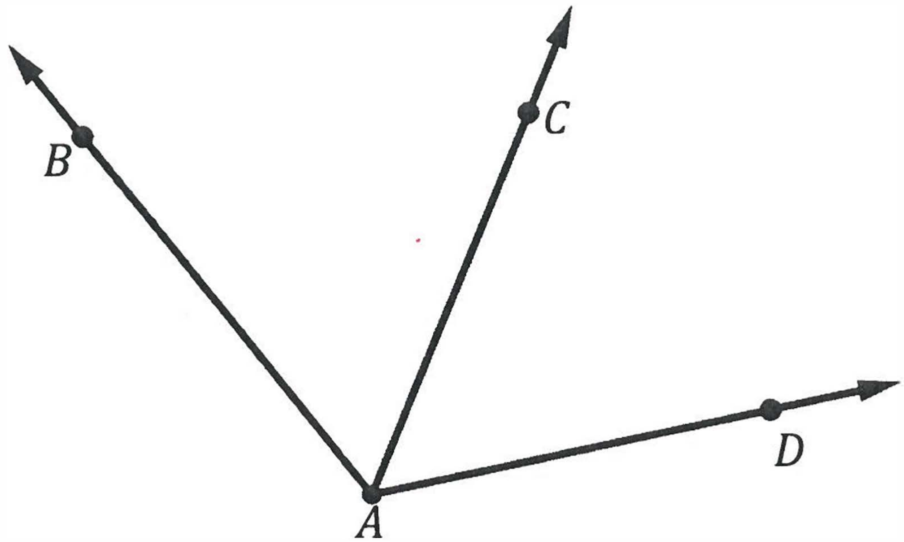
If $m\angle Z = m\angle A + m\angle C$, then $\angle Z$ is

- ○ acute
- ○ obtuse
- ● right
- ○ straight

.

Section 2 – Topic 3
Angle Pairs – Part 1

Consider the following figure that presents an ***angle pair***.

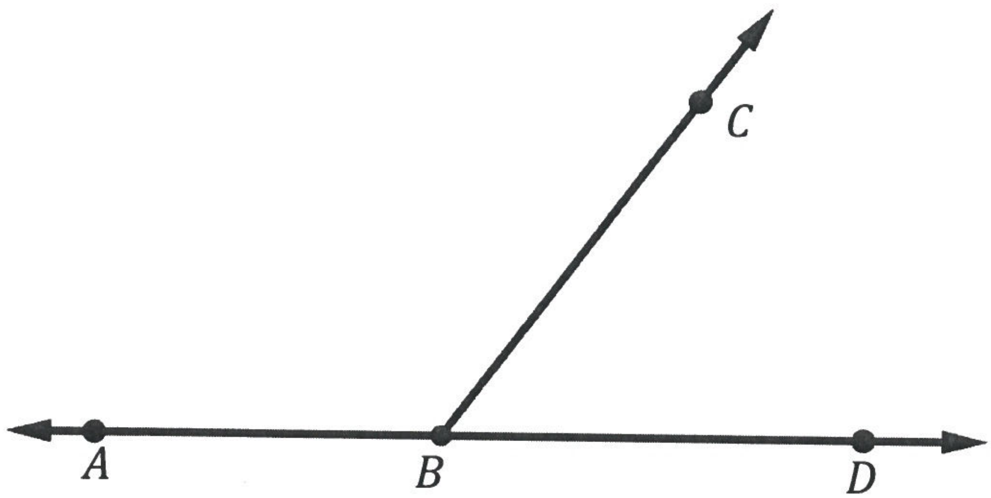


What common ray do $\angle BAC$ and $\angle CAD$ share?

$\overrightarrow{AC}$

Because these angle pairs share a ray, they are called ___adjacent___ angles.

Consider the following figure of ***adjacent angles***.



What observations can you make about the figure?

$\angle ABC$ is adjacent to $\angle CBD$.

These adjacent angles are called a linear pair.
Together, the angles form a straight angle.

What is the measure of a straight angle?

180°

*What is the measure of the sum of a **linear pair**?*

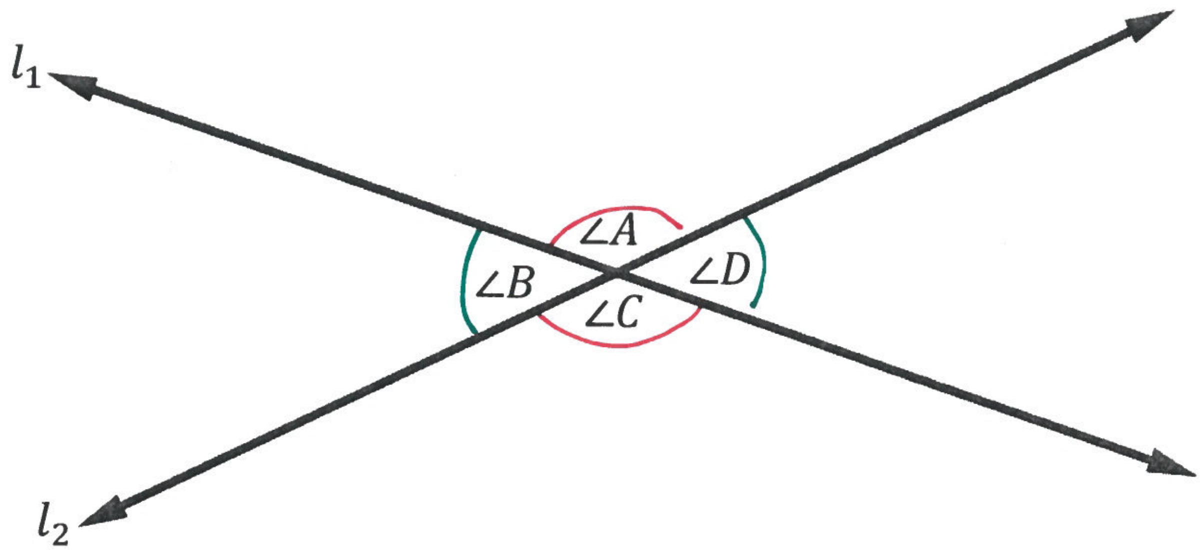
180°

TAKE NOTE! *Postulates & Theorems*

Linear Pair Postulate

If two positive angles form a linear pair, then they are supplementary.

Consider the figure below of angle pairs.



What observations can you make about $\angle A$ and $\angle C$?

Opposite angles

What observations can you make about $\angle B$ and $\angle D$?

Opposite angles

$\angle A$ and $\angle C$ form what we call a pair of vertical angles.

What angle pair(s) form(s) a set of ***vertical angles****?*

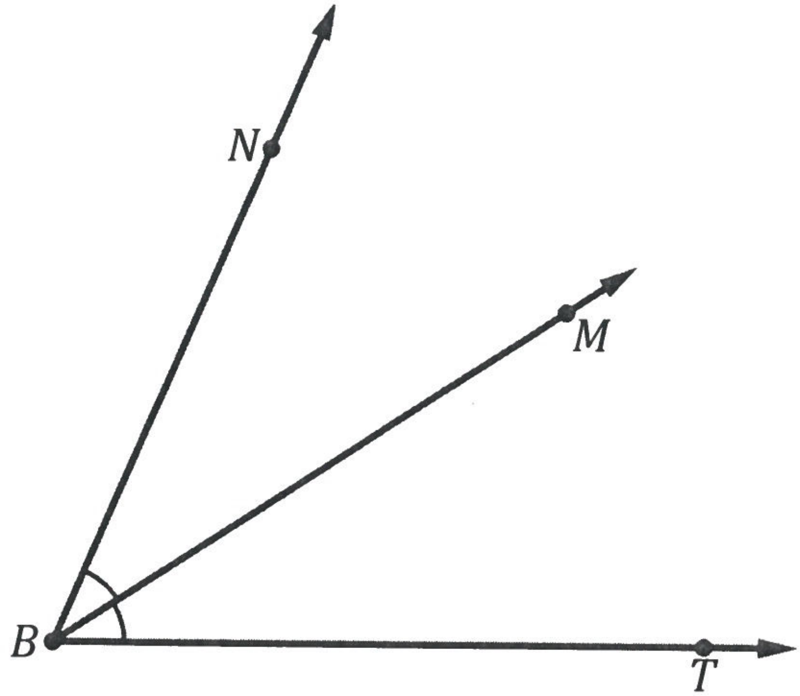
$\angle A$ and $\angle C$; $\angle B$ and $\angle D$

TAKE NOTE! Postulates & Theorems

Vertical Angles Theorem

If two angles are vertical angles, then they have equal measures.

Consider the figure below.



What observations can you make about the figure?

$\angle NBM$ is adjacent to $\angle MBT$. Both angles have the same arc. Are these angles the same?

*We call $\overrightarrow{BM}$ an **angle bisector**.*

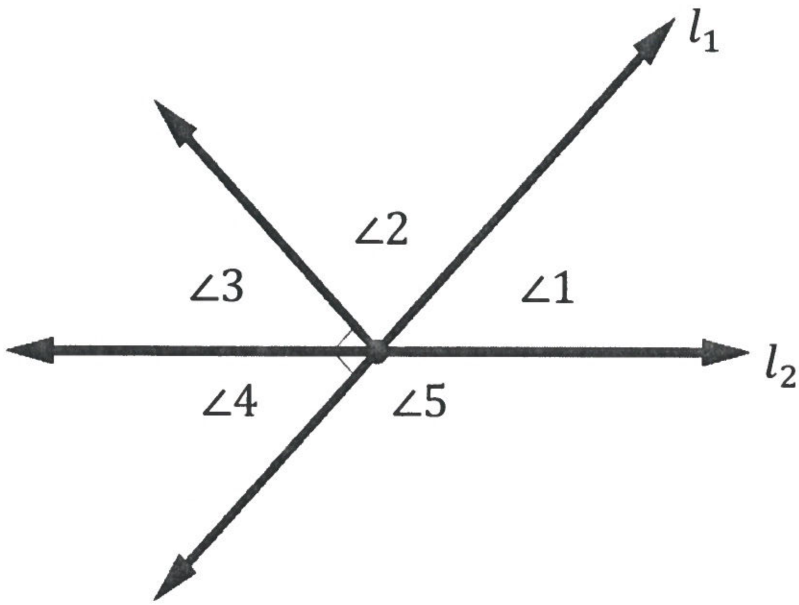
Make a conjecture as to why $\overrightarrow{BM}$ is called an angle bisector.

$\overrightarrow{BM}$ divides $\angle NBT$ in two congruent angles $\angle NBM$ and $\angle MBT$.

Yes!

Let's Practice!

1. Consider the figure below.



Complete the following statements:

- $\angle 1$ and $\angle 4$ are vertical angles.
- $\angle 1$ and $\angle 2$ are adjacent angles.
- $\angle 3$ and $\angle 4$ are adjacent angles and complimentary angles.
- $\angle 4$ and $\angle 5$ are adjacent angles and supplementary angles. They also form a linear pair.

2. If $\angle ACB$ and $\angle ACE$ are linear pairs, and $m\angle ACB = 5x + 25$ and $m\angle ACE = 2x + 29$, then

a. Determine $m\angle ACB + m\angle ACE$.

Add to 180°

b. Determine the measures of $m\angle ACB$ and $m\angle ACE$.

$(5x+25)+(2x+29)=180$

$5x+25+2x+29=180$

$7x+54=180$

$-54 \quad -54$

$7x=126$

$\frac{7x}{7}=\frac{126}{7}$

$x=18$

$m\angle ACB = 5(18)+25$
$= 90+25$
$= 115°$

$m\angle ACE = 2(18)+29$
$= 36+29$
$= 65°$

3. If $\angle MFG$ and $\angle EFN$ are vertical angles, and $m\angle MFG = 7x - 18$ and $m\angle EFN = 5x + 10$, then

a. What can we say about $\angle MFG$ and $\angle EFN$ that will help us determine their measures?

Equal measures (congruent angles)

b. Determine the measures of $\angle MFG$ and $\angle EFN$.

$(7x-18)=(5x+10)$

$+18 \quad +18$

$7x=5x+28$

$-5x \quad -5x$

$\frac{2x}{2}=\frac{28}{2}$

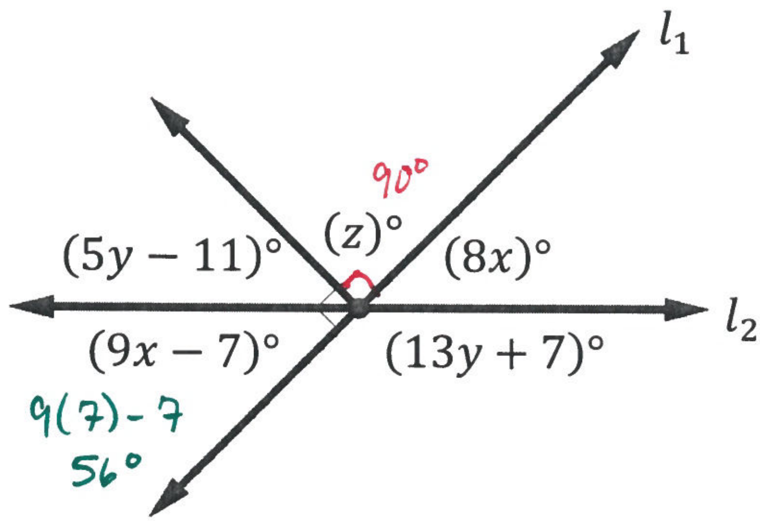
$x=14$

$m\angle MFG = 7(14)-18$
$= 98-18$
$= 80°$

$m\angle EFN = 5(14)+10$
$= 70+10$
$= 80°$

Try It!

4. Consider the figure below.



Angles measures are represented by algebraic expressions. Find the value of x, y, and z.

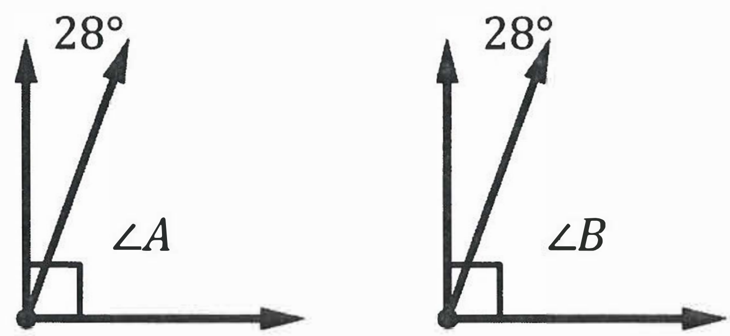
$8x = 9x - 7$
$-9x \quad -9x$
$\frac{-x}{-1} = \frac{+7}{-1}$
$\boxed{x = 7}$

$5y - 11 + 56 = 90$
$5y + 45 = 90$
$-45 \quad -45$
$\frac{5y}{5} = \frac{45}{5}$
$\boxed{y = 9}$

$z = 90°$
Linear Pair Postulate

Section 2 – Topic 4
Angle Pairs – Part 2

Consider the figure below.



What can you observe about $\angle A$ and $\angle B$?

Congruent

TAKE NOTE! Postulates & Theorems

Congruent Complements Theorem

If $\angle A$ and $\angle B$ are complements of the same angle, then $\angle A$ and $\angle B$ are congruent.

Consider the figures below.

What can you observe about $\angle A$ and $\angle B$?

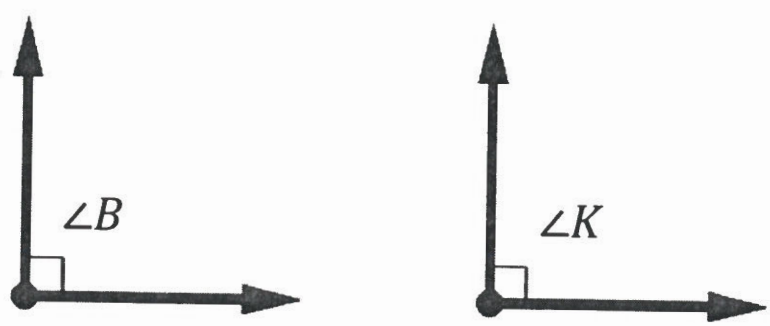
Congruent

TAKE NOTE! Postulates & Theorems

Congruent Supplements Theorem

If $\angle A$ and $\angle B$ are supplements of the same angle, then $\angle A$ and $\angle B$ are congruent.

Consider the figure below.



What can you observe about $\angle B$ and $\angle K$?

Congruent

TAKE NOTE! Postulates & Theorems

Right Angles Theorem

All right angles are congruent.

Let's Practice!

1. The measure of an angle is four times greater than its complement. What is the measure of the larger angle?

x and $4x$

$4x + x = 90$

$\frac{\cancel{5}x}{\cancel{5}} = \frac{90}{5}$

$x = 18$

x $4(18)$

$18°$ $72°$

Try It!

2. $\angle X$ and $\angle Y$ are supplementary (180°). One angle measures 5 times the other angle. What is the complement (90°) of the smaller angle?

$x + 5x = 180$

$\frac{\cancel{6}x}{\cancel{6}} = \frac{180}{6}$

$x = 30°$

x $5(30)$

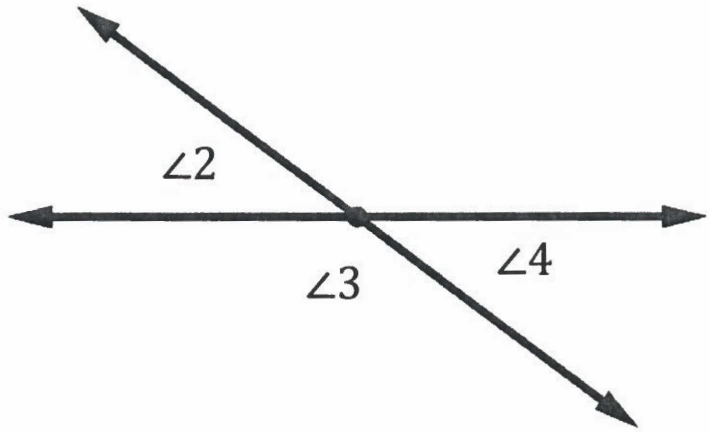
$30°$ $150°$

$\downarrow$

$90 - 30 = 60°$

Let's Practice!

3. Consider the figure below.



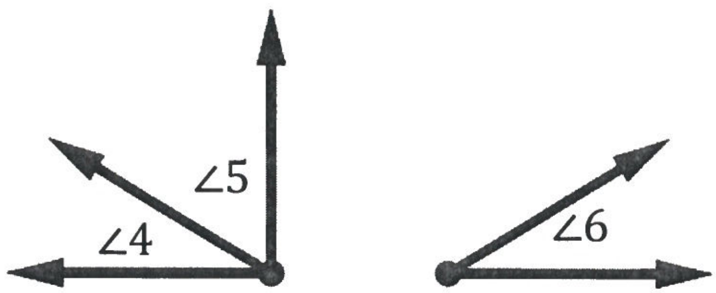
Given: ∠2 and ∠3 are a linear pair. **Prove:** ∠2 ≅ ∠4
∠3 and ∠4 are a linear pair.

Complete reasons 2 and 3 in the chart below.

Statements	Reasons
1. ∠2 and ∠3 are a linear pair. ∠3 and ∠4 are a linear pair.	**1.** Given
2. ∠2 and ∠3 are supplementary. ∠3 and ∠4 are supplementary.	**2.** Linear Pair Postulate
3. ∠2 ≅ ∠4	**3.** Congruent Supplements Theorem

Try It!

4. Consider the figure below.



Given: ∠5 and ∠6 are complementary.
$m\angle 4 + m\angle 5 = 90°$

Prove:
$\angle 6 \cong \angle 4$

Complete the chart below.

Statements	Reasons
1. ∠5 and ∠6 are complementary	**1.** Given
2. $m\angle 4 + m\angle 5 = 90°$	**2.** Given
3. ∠4 and ∠5 are complementary	**3.** Definition of complementary angles
4. $\angle 6 \cong \angle 4$	**4.** Congruent complements theorem.

BEAT THE TEST!

1. $\angle LMN$ and $\angle PML$ are linear pairs, $m\angle LMN = 7x - 3$ and $m\angle PML = 13x + 3$.

$(7x-3)+(13x+3)=180$

$7x-3+13x+3=180$

$\frac{20x}{20}=\frac{180}{20}$

$x=9$

Part A: $m\angle LMN =$ $7(9)-3$

$=63-3$

$=60°$

Part B: $m\angle PML =$ $13(9)+3=117+3$

$=120°$

Part C: If $\angle PMR$ and $\angle LMN$ form a vertical pair and $m\angle PMR = 5y + 4$, find the value of y?

$\cong$

$m\angle PMR = m\angle LMN$

$5y+4=60$

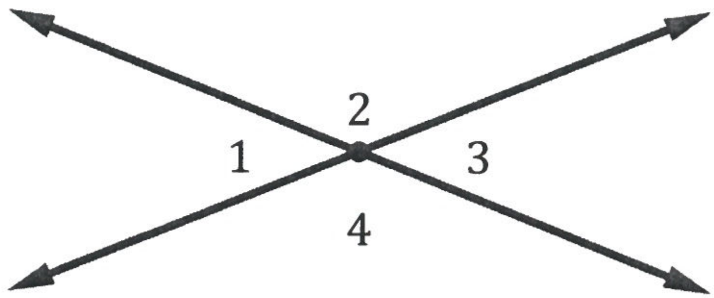
$-4 \quad -4$

$\frac{5y}{5}=\frac{56}{5}$

$y=\frac{56}{5}$

$\boxed{y=11.2}$

2. Consider the figure below.



Given: ∠1 and ∠2 form a linear pair.
∠1 and ∠4 form a linear pair.

Prove: The Vertical Angle Theorem

Use the bank of reasons below to complete the table.

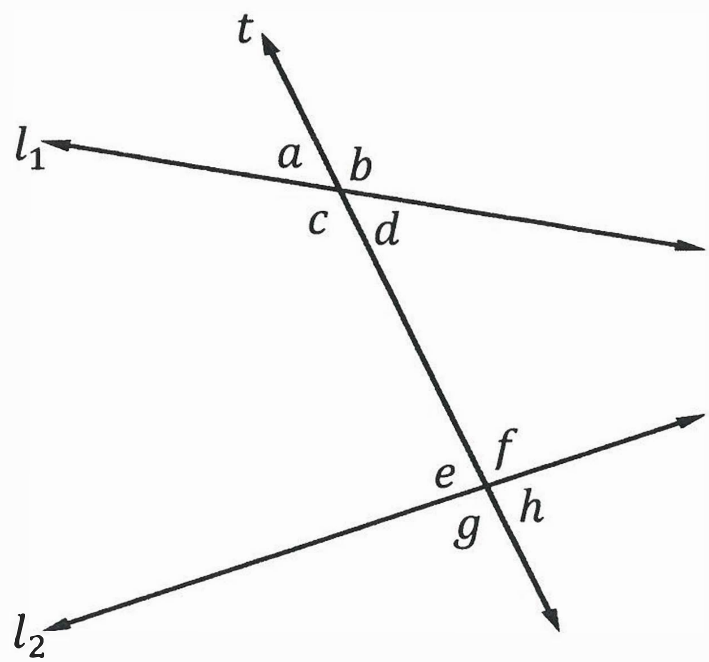
~~Congruent Supplement Theorem~~	Right Angles Theorem
Congruent Complement Theorem	~~Linear Pair Postulate~~

Statements	Reasons
1. ∠1 and ∠2 are linear pairs. ∠1 and ∠4 are linear pairs.	**1.** Given
2. ∠1 and ∠2 are supplementary. ∠1 and ∠4 are supplementary.	**2.** Linear Pair Postulate
3. ∠2 ≅ ∠4	**3.** Congruent Supplements theorem

Section 2 – Topic 5
Special Types of Angle Pairs Formed by Transversals and Non-Parallel Lines

Many geometry problems involve the intersection of three or more lines.

Consider the figure below.



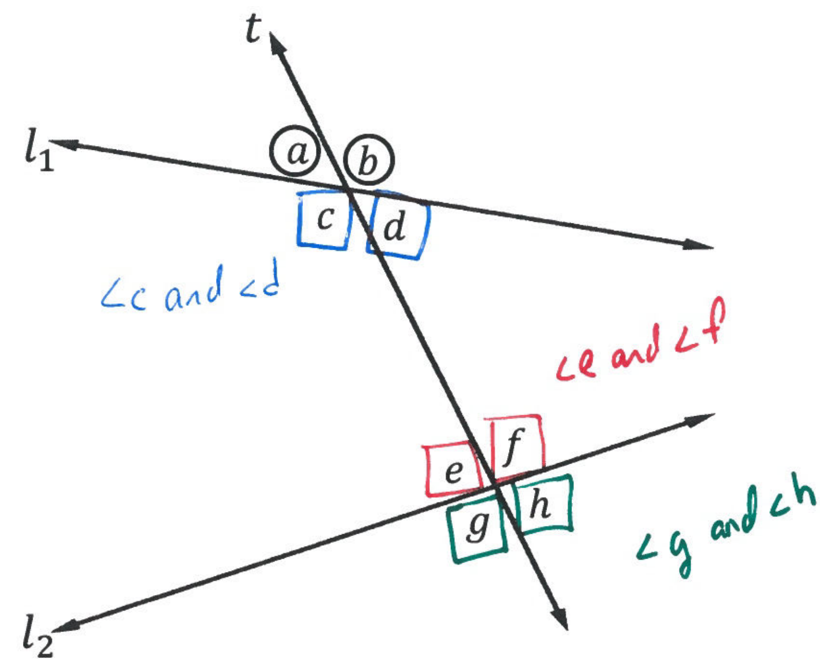
What observations can you make about the figure?

Many linear pairs and vertical angles.

- Lines l_1 and l_2 are crossed by line t.
- Line t is called the transversal, because it intersects two other lines (l_1 and l_2).
- The intersection of line t with l_1 and l_2 forms eight angles.

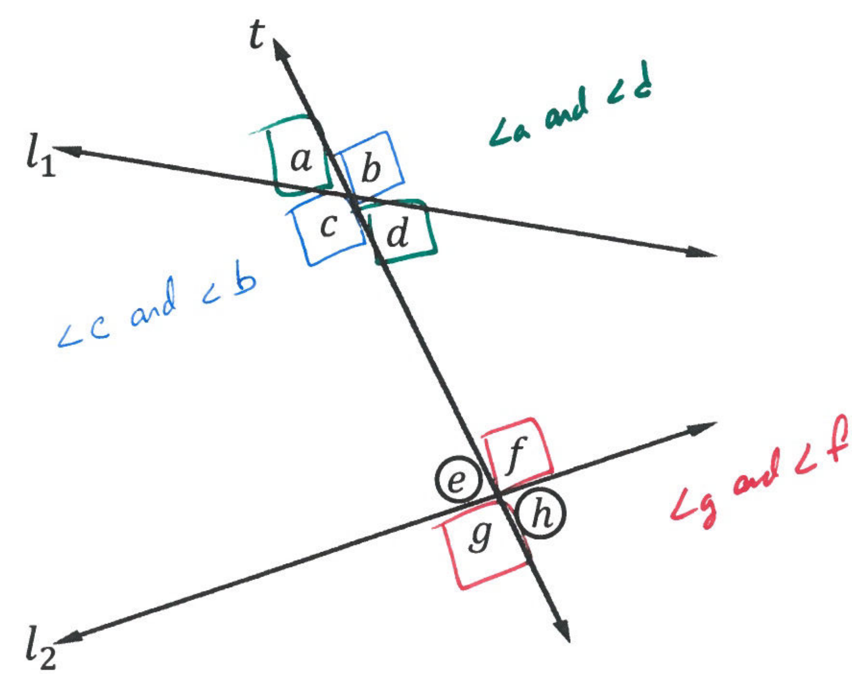
Identify angles made by transversals

Consider the figure below. $\angle a$ and $\angle b$ form a ***linear pair***.



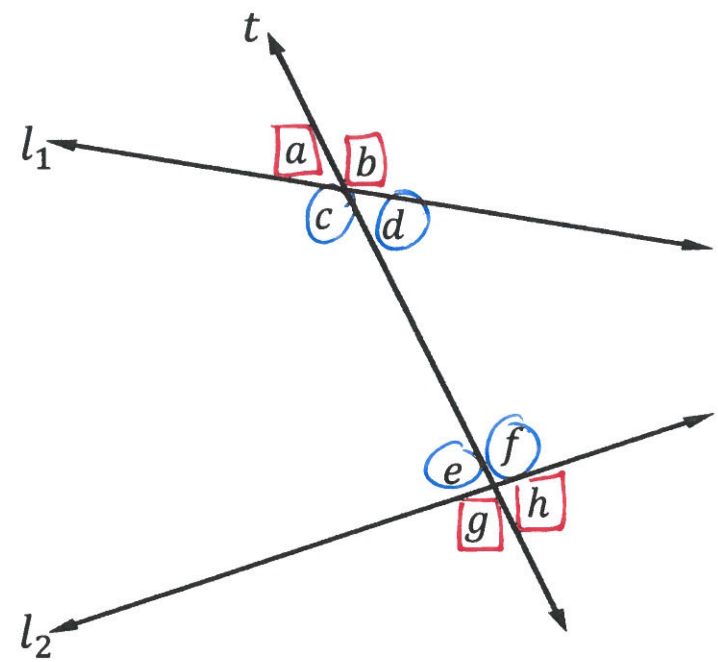
Box and name the other linear pairs in the figure.

Consider the figure below. $\angle e$ and $\angle h$ are ***vertical angles***.



Box and name the other pairs of vertical angles in the figure.

Consider the figure below.



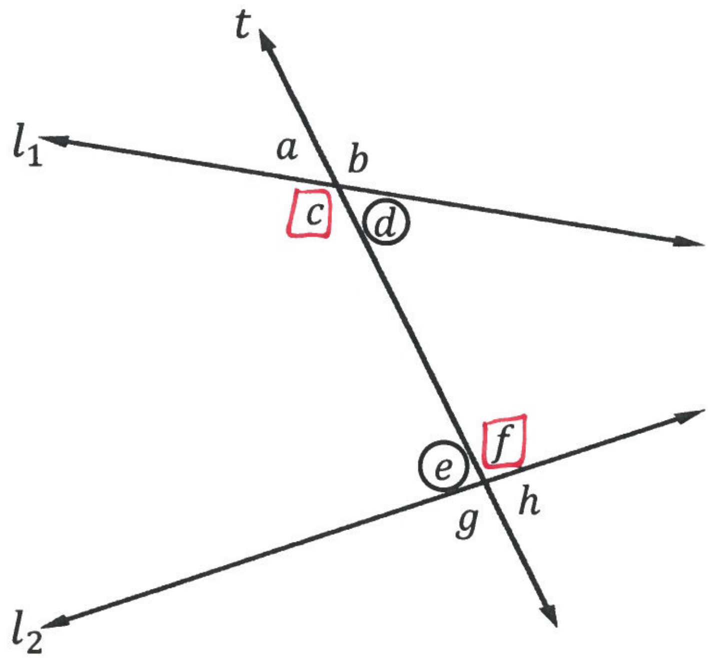
Which part of the figure do you think would be considered the interior? Draw a circle around the interior angles in the figure. Justify your answer.

Between (inside) lines l_1 and l_2

Which part of the figure do you think would be considered the exterior? Draw a box around the exterior angles in the figure. Justify your answer.

Outside of lines l_1 and l_2

Consider the figure below. $\angle d$ and $\angle e$ are ***alternate interior angles***.

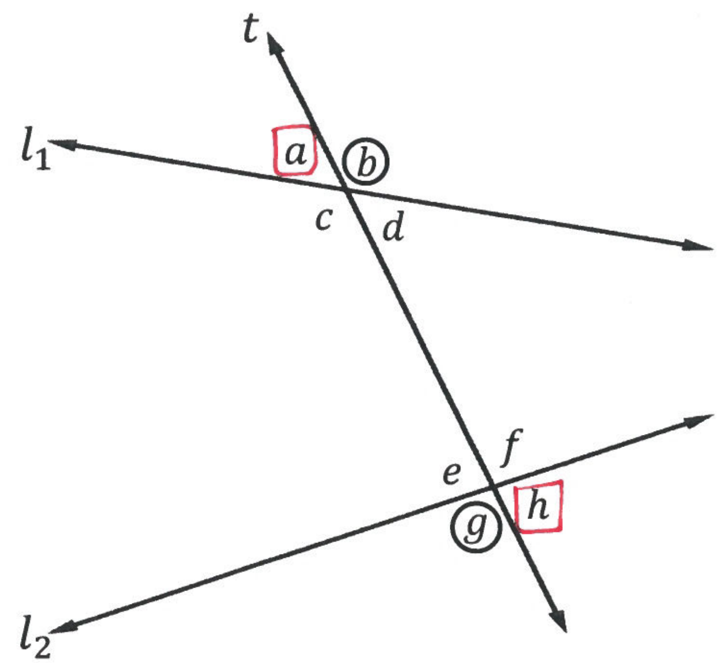


- The angles are in the interior region of the lines l_1 and l_2.
- The angles are on opposite sides of the transversal.

alternate

Draw a box around the other pair of alternate interior angles in the figure.

Consider the figure below. $\angle b$ and $\angle g$ are ***alternate exterior angles***.

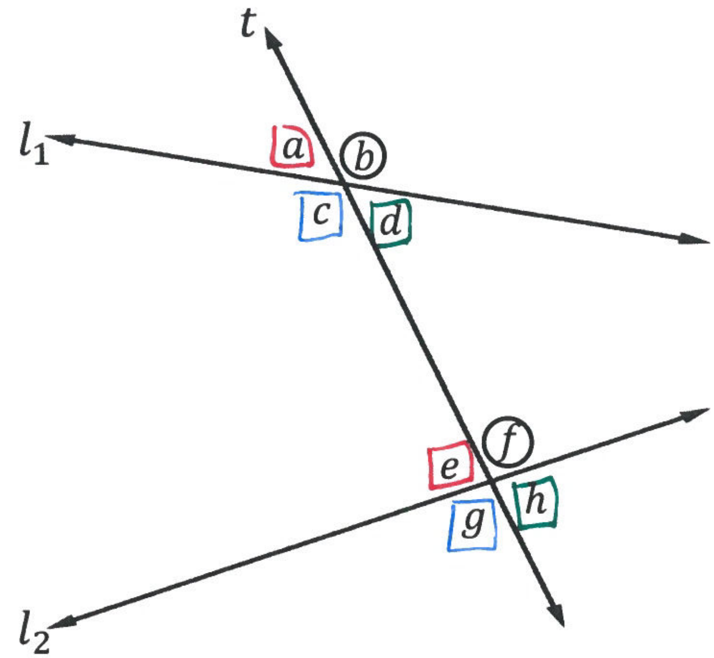


- ➢ The angles are in the exterior region of lines l_1 and l_2.
- ➢ The angles are on opposite sides of the transversal.

alternate

Draw a box around the other pair of alternate exterior angles in the figure.

Consider the figure below. $\angle b$ and $\angle f$ are ***corresponding angles***.



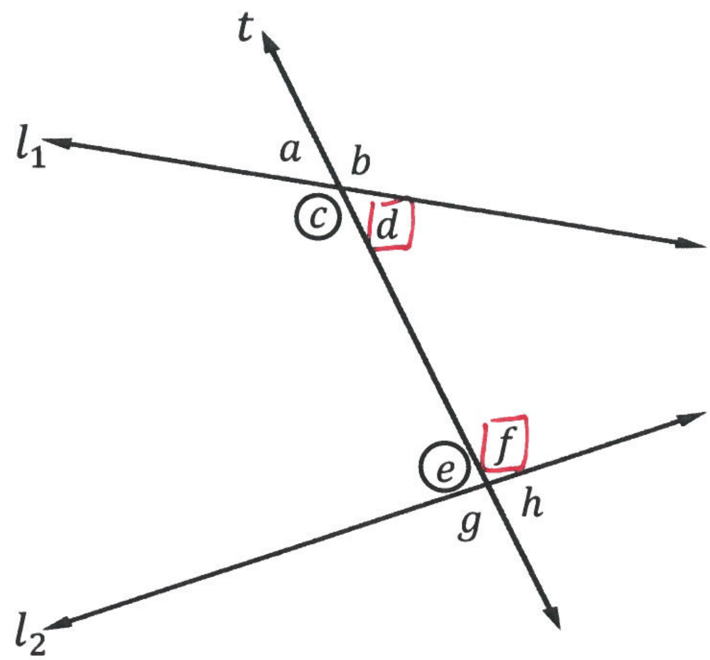
- The angles have distinct vertex points.

- The angles lie on the same side of the transversal.
  corresponding

- One angle is in the interior region of lines l_1 and l_2. The other angle is in the exterior region of lines l_1 and l_2.

Draw a box around the other pairs of corresponding angles in the figure and name them below.

Consider the figure below. $\angle c$ and $\angle e$ are **consecutive** or ***same-side interior angles***.



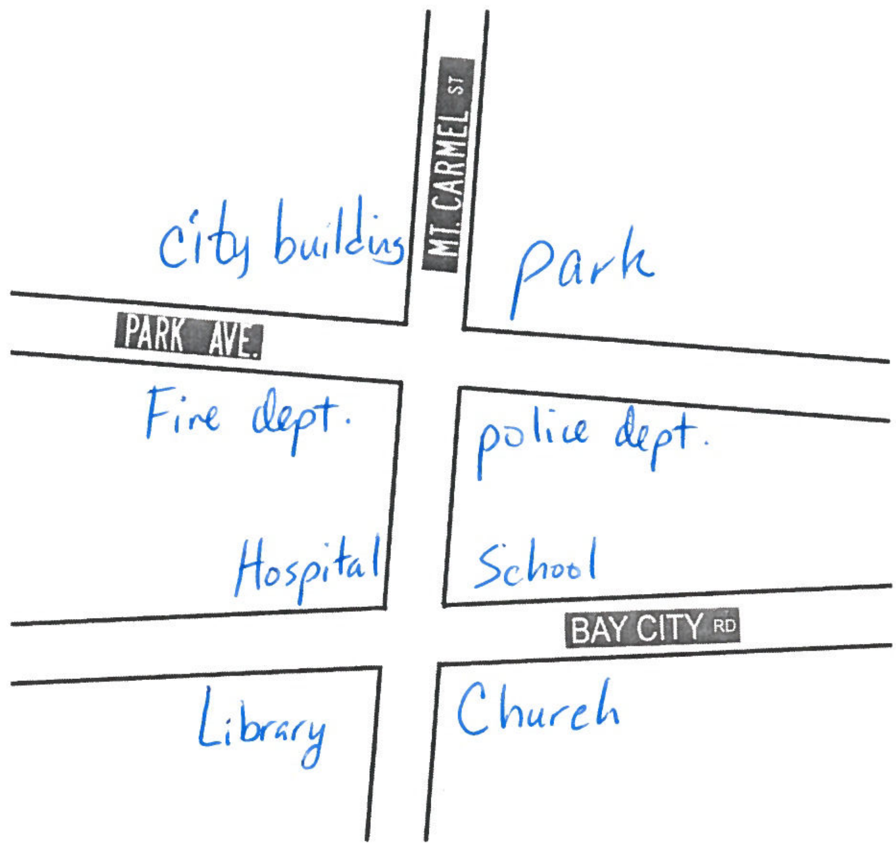
- The angles have distinct vertex points.
- The angles lie on the same side of the transversal.
- Both angles are in the interior region of lines l_1 and l_2.

consecutive

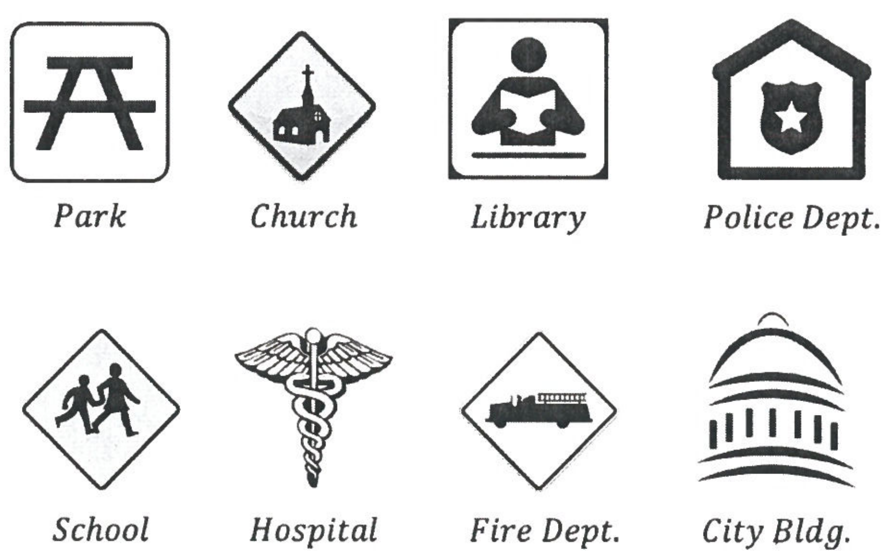
Draw a box around the other pair of consecutive interior angles.

Let's Practice!

1. On the figure below, Park Ave. and Bay City Rd. are non-parallel lines crossed by transversal Mt. Carmel. St.



The city hired GeoNat Road Svc. to plan where certain buildings will be constructed and located on the map.

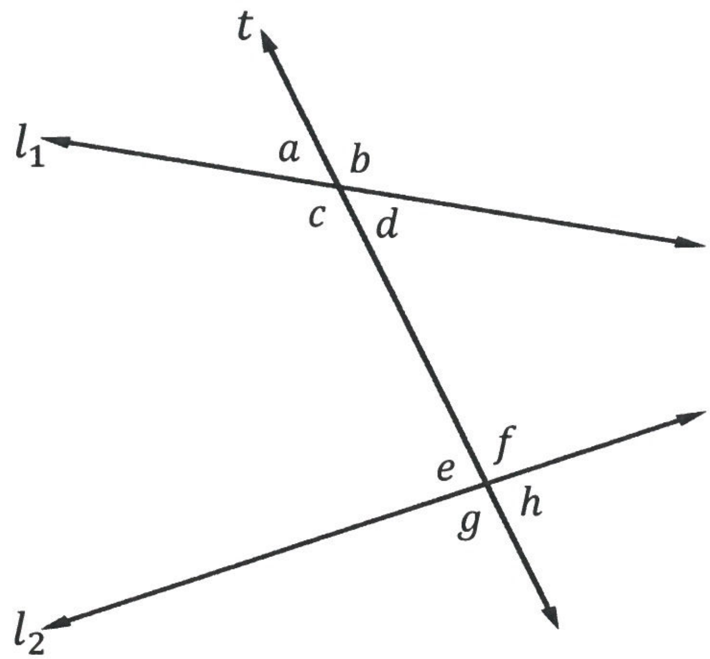


Position the buildings on the map by meeting the following conditions:

- The park and the city building form a linear pair.
- The city building and the police department are at vertical angles.
- The police department and the hospital are at alternate interior angles.
- The hospital and the fire department are at consecutive interior angles.
- The school is at a corresponding angle with the park and a consecutive interior angle to the police department.
- The library and the park are at alternate exterior angles.
- The church is at an exterior angle and it forms a linear pair with both the library and the school.

Try It!

2. Consider the figure below.



Which of the following statements is true?

Ⓐ $\angle a$ and $\angle e$ lie on the same side of the transversal and one angle is interior and the other is exterior, so they are corresponding angles.

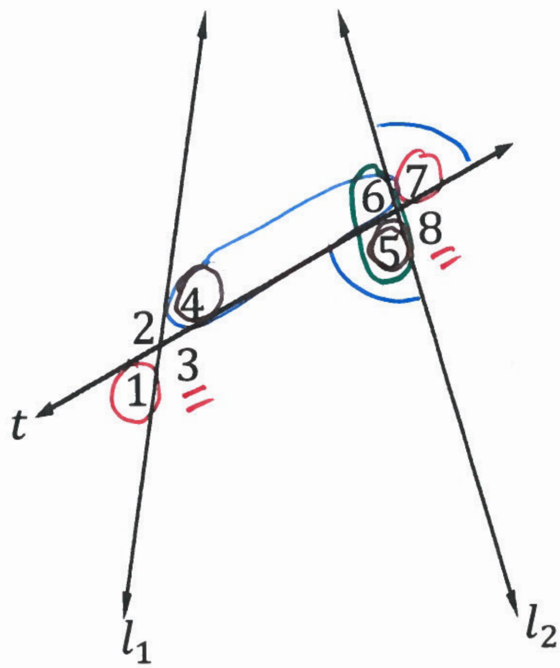
Ⓑ If $\angle b$ and $\angle h$ are on the exterior opposite sides of the transversal, so they are alternate exterior angles.

Ⓒ If $\angle b$ and $\angle c$ are adjacent angles lying on the same side of the transversal, then they are same-side/consecutive interior angles.

Ⓓ If $\angle b, \angle c, \angle f$ and $\angle g$ are between the non-parallel lines, then they are interior angles.

BEAT THE TEST!

1. Consider the figure below.

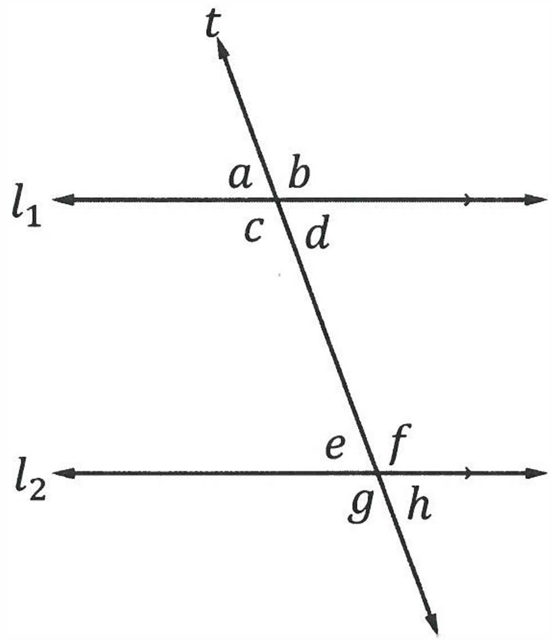


Match the angles on the left with their corresponding names on the right. Write the letter of the most appropriate answer beside each angle pair below.

E	$\angle 1$ and $\angle 7$	~~A.~~	Alternate Interior Angles
F	$\angle 5$ and $\angle 6$	~~B.~~	Consecutive Angles
B	$\angle 4$ and $\angle 6$	~~C.~~	Corresponding Angles
D	$\angle 5$ and $\angle 7$	~~D.~~	Vertical Angles
A	$\angle 4$ and $\angle 5$	~~E.~~	Alternate Exterior Angles
C	$\angle 3$ and $\angle 8$	~~F.~~	Linear Pair

Section 2 – Topic 6
Special Types of Angle Pairs Formed by Transversals and Parallel Lines – Part 1

Consider the following figure of a transversal crossing two parallel lines.



Name the acute angles in the above figure.

$\angle a$, $\angle d$, $\angle e$ and $\angle h$

Name the obtuse angles in the above figure.

$\angle b$, $\angle c$, $\angle f$ and $\angle g$

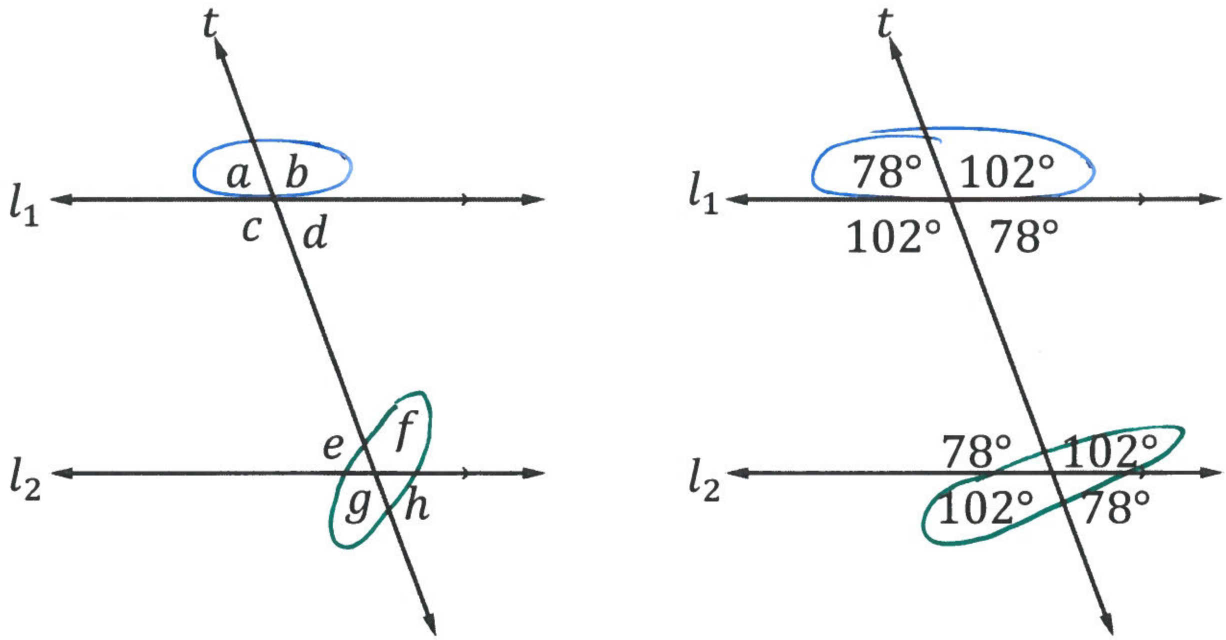
Which angles are congruent? Justify your answer.

All pairs of vertical angles: Vertical Angle Theorem

Which angles are supplementary? Justify your answer.

All pairs of linear pairs: Linear Pair Postulate.

Consider the following figures of transversal t crossing parallel lines, l_1 and l_2.



Identify an example of the ***Linear Pair Postulate****. Use the figure above to justify your answer.*

∠a and ∠b form a linear pair; so they are supplementary.

Consider → 78° + 102° = 180°

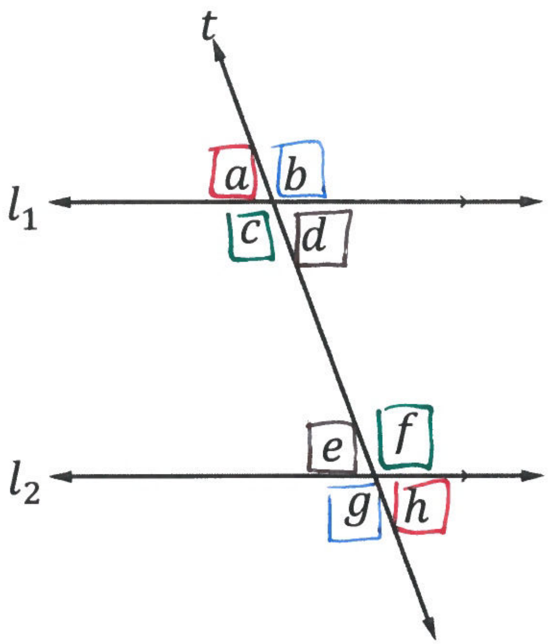
Identify an example of the ***Vertical Angles Theorem****. Use the figure above to justify your answer.*

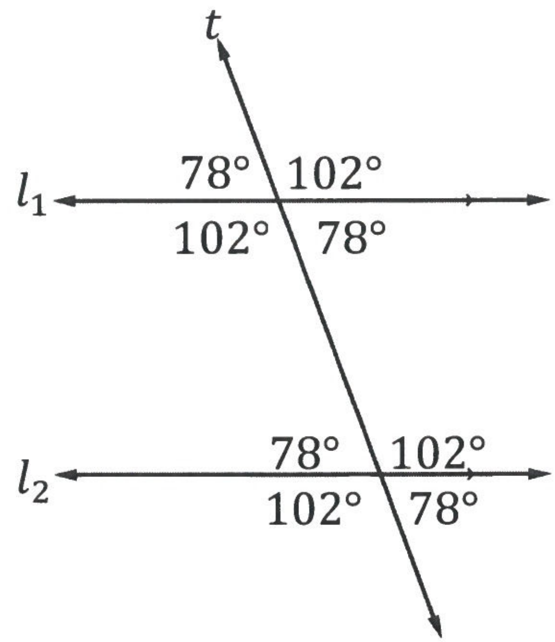
∠f and ∠g are vertical angles and considering that 102° = 102°, then it proves they are congruent.

Make a list of the interior and the exterior angles. What can you say about these angles?

Interior: ∠c | ∠d | ∠e | ∠f
Exterior: ∠a | ∠b | ∠g | ∠h

Interior angles total 360°.
Also, exterior angles total 360°.





*Identify each of the **alternate interior angles** in the above figures and determine the angles' measures.* $m\angle c = m\angle f = 102°$
$m\angle d = m\angle e = 78°$

TAKE NOTE! *Postulates & Theorems*

<u>Alternate Interior Angles Theorem:</u>

If two parallel lines are cut by a transversal, the alternate interior angles are congruent.

<u>Converse of the Alternate Interior Angles Theorem:</u>

If two lines are cut by a transversal and the alternate interior angles are congruent, the lines are parallel.

*Identify the **alternate exterior angles** in the above figures and determine the angles' measures.* $m\angle b = m\angle g = 102°$
$m\angle a = m\angle h = 78°$

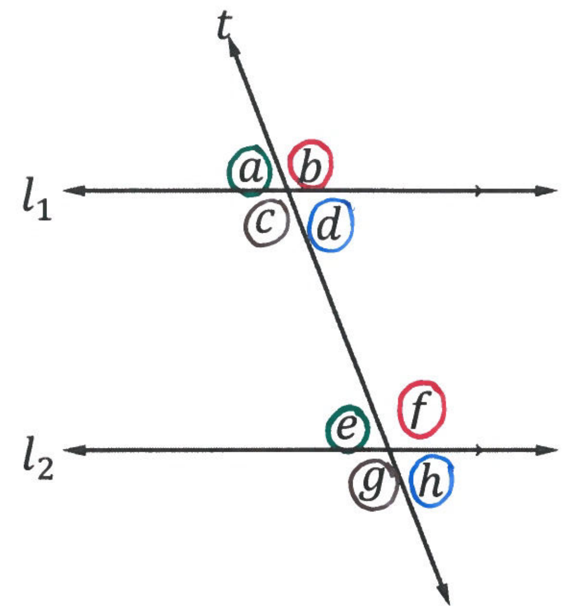
TAKE NOTE! *Postulates & Theorems*

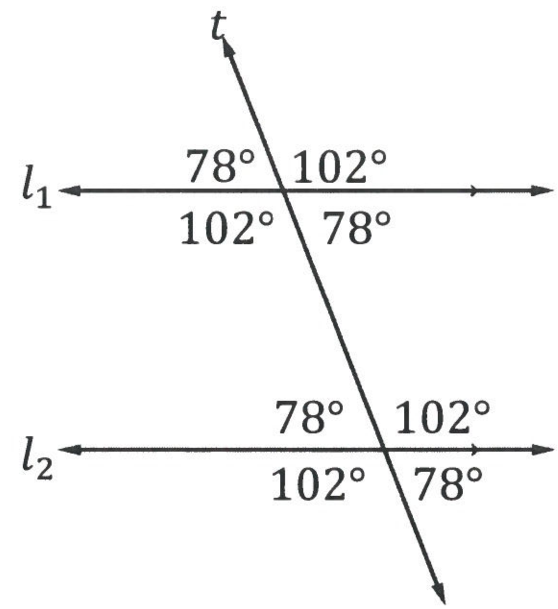
<u>Alternate Exterior Angles Theorem:</u>

If two parallel lines are cut by a transversal, the alternate exterior angles are congruent.

<u>Converse of the Alternate Exterior Angles Theorem:</u>

If two lines are cut by a transversal and the alternate exterior angles are congruent, the lines are parallel.





Identify the ***corresponding angles*** *in the above figures. What does each angle measure?*

$m\angle a = m\angle e = 78°$

$m\angle c = m\angle g = 102°$

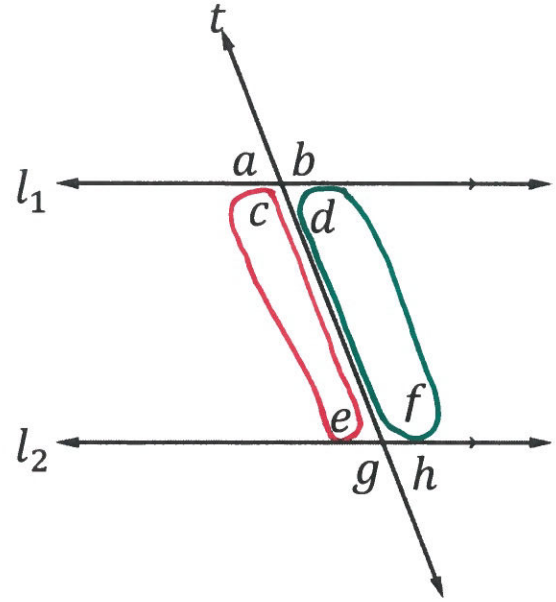
$m\angle b = m\angle f = 102°$

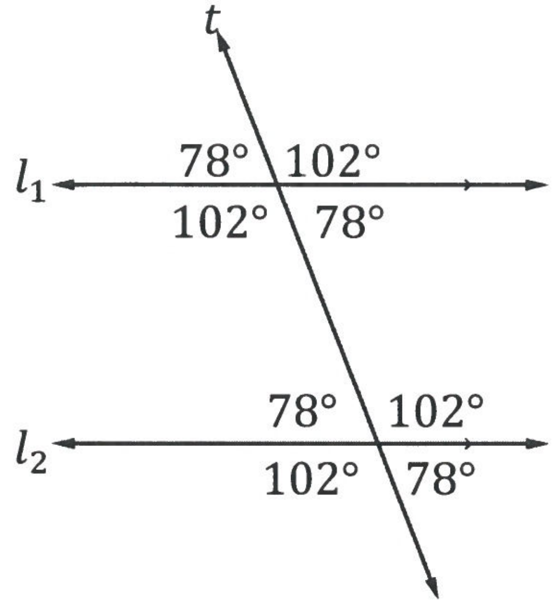
$m\angle d = m\angle h = 78°$

TAKE NOTE!
Postulates & Theorems

<u>Corresponding Angles Theorem:</u>
If two parallel lines are cut by a transversal, the corresponding angles are congruent.

<u>Converse of the Corresponding Angles Theorem:</u>
If two lines are cut by a transversal and the corresponding angles are congruent, the lines are parallel.





*Identify the **same-side/consecutive angles** in the above figures. What does each angle measure?*

$m\angle c + m\angle e = 180°$
$102° + 78° = 180°$

$m\angle d + m\angle f = 180°$
$78° + 102° = 180°$

TAKE NOTE! *Postulates & Theorems*

Same-side Consecutive Angles Theorem:

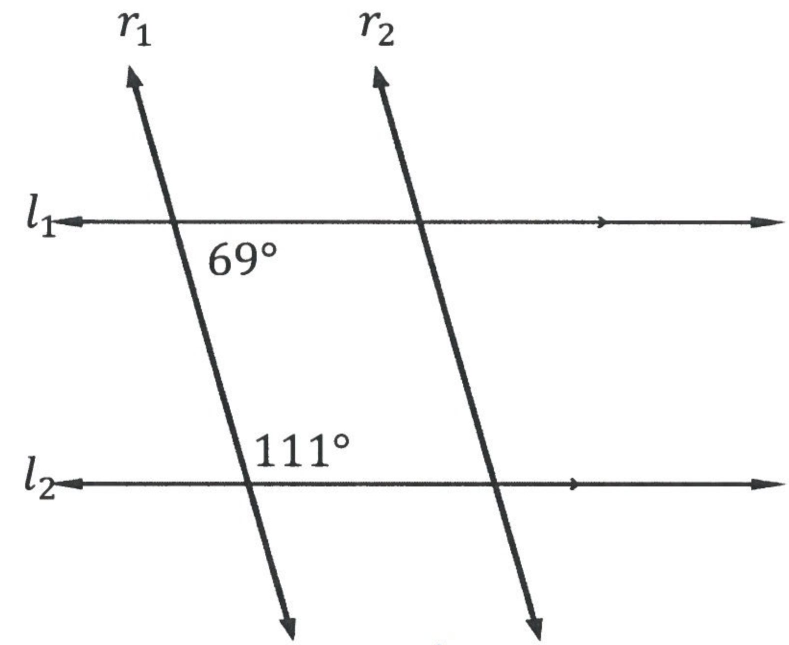
If two parallel lines are cut by a transversal, the interior angles on the same side of the transversal are supplementary.

Converse of the Same-side Consecutive Angles Theorem:

If two lines are cut by a transversal and the interior angles on the same side of the transversal are supplementary, the lines are parallel.

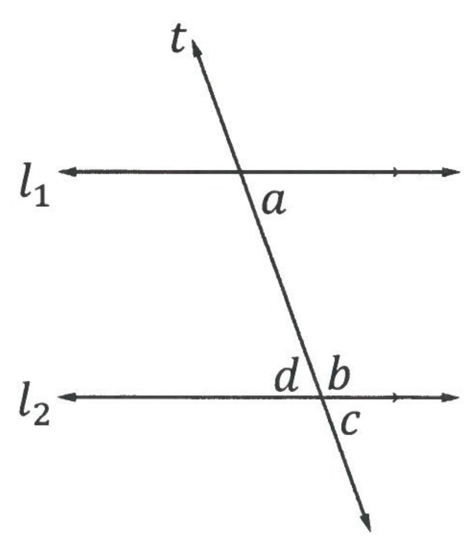
Let's Practice!

1. Which lines of the following segments are parallel? Circle the appropriate answer, and justify your answer.



Ⓐ ~~r_1 and r_2~~ not enough information
Ⓑ l_1 and l_2 $69° + 111° = 180°$ Converse of same side/consecutive angles theorem
Ⓒ ~~r_1 and l_2~~ intersect
Ⓓ ~~l_1 and r_2~~ intersect

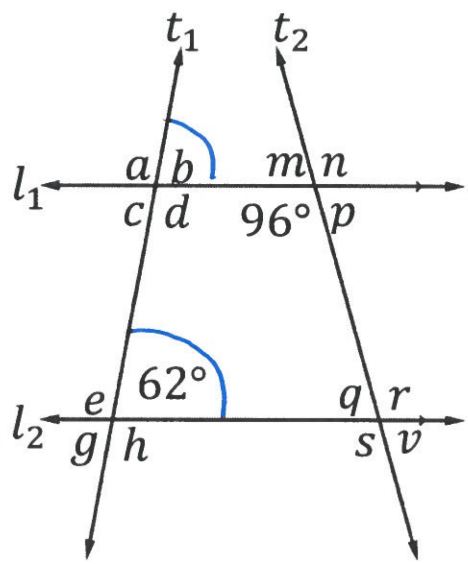
2. Which of the following is a condition for the figure below that will ***not*** prove $l_1 \parallel l_2$?



Ⓐ $\angle a \cong \angle c$ ✓ corresponding
Ⓑ $\angle b + \angle d = 180$ NO
Ⓒ $\angle a \cong \angle d$ ✓ alternate interior
Ⓓ $\angle a + \angle b = 180$ ✓ consecutive

Try It!

3. Consider the figure below, where l_1 and l_2 are parallel and cut by transversals t_1 and t_2. Find the values of a, b and v.



m∠b = 62°

Corresponding Angles Theorem

m∠a = 180° − 62°

m∠a = 118°

Linear Pairs Postulate

m∠s = 96°

Corresponding Angles Theorem

m∠s + m∠v = 180°

Linear Pairs Postulate

$\cancel{96°}$ + m∠v = 180°

$\cancel{-96°}$ −96°

m∠v = 84°

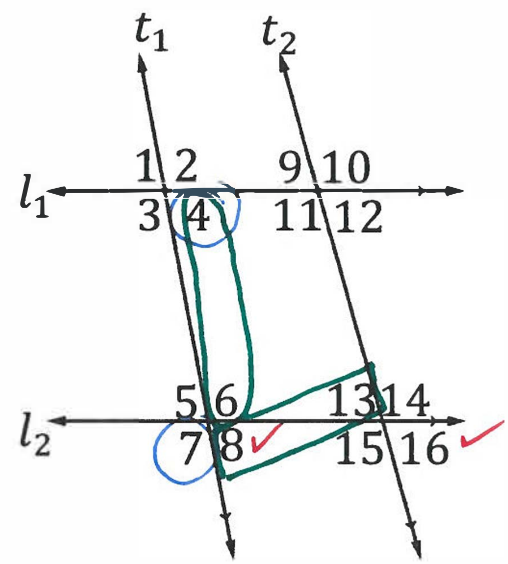
Section 2 – Topic 7
Special Types of Angle Pairs Formed by Transversals and Parallel Lines – Part 2

Let's Practice!

1. Complete the chart below using the following information.

   Given:
   $\angle 4$ and $\angle 7$ are supplementary. $\angle 8$ and $\angle 16$ are congruent.

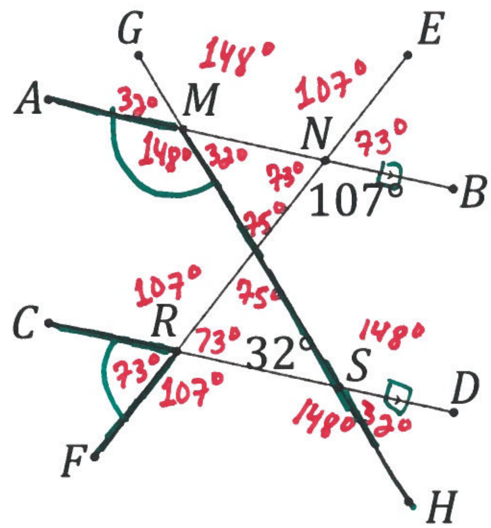
   Prove: $l_1 \parallel l_2$ and $t_1 \parallel t_2$



Statements	Reasons
1. ∠4 and ∠7 are supplementary.	**1.** Given
2. ∠8 and ∠16 are congruent.	**2.** Given
3. $\angle 7 \cong \angle 6$; $\angle 13 \cong \angle 16$	**3.** Vertical Angle Theorem
4. ∠4 and ∠6 are supplementary and ∠8 and ∠13 ⇒ congruent.	**4.** Substitution
5. $l_1 \parallel l_2$	**5.** Converse of same-side consecutive angles theorem
6. $t_1 \parallel t_2$	**6.** Converse of the alternate interior angles theorem.

Try It!

2. Consider the figure below. Find the measures of $\angle AMS$ and $\angle CRF$, and justify your answers.



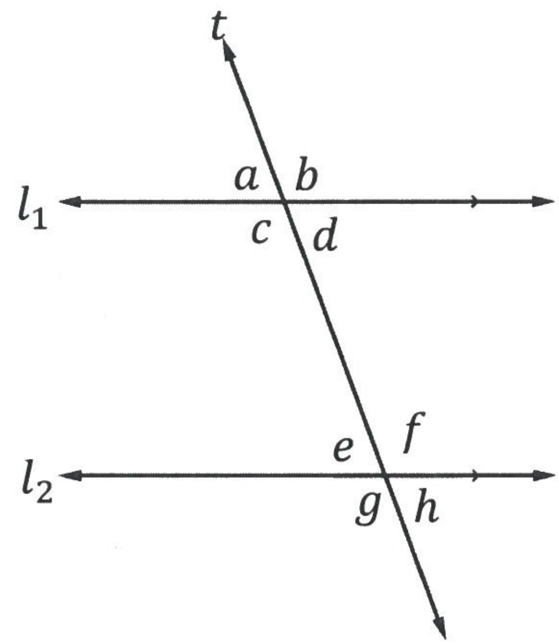
$m\angle AMS = 148°$

$m\angle CRF = 73°$

3. Complete the chart below using the following information.

Given: $l_1 \parallel l_2$

Prove: $m\angle a + m\angle g = 180°$



Statements	Reasons
1. $l_1 \parallel l_2$	**1.** Given
2. $\angle a$ and $\angle c$ supplementary	**2.** Linear Pair Postulate
3. $m\angle a + m\angle c = 180°$	**3.** Definition of Supplementary
4. $\angle c \cong \angle g$	**4.** Corresponding Angles Theorem
5. $m\angle c = m\angle g$	**5.** Definition of Congruent
6. $m\angle a + m\angle g = 180°$	**6.** Substitution

BEAT THE TEST!

1. Consider the figure below in which $l_1 \parallel l_2$, $m\angle a = 13y$, $m\angle b = 31y + 4$, $m\angle r = 30x + 40$, and $m\angle s = 130x - 160$.

$m\angle a = 13y = 13(4) = 52°$

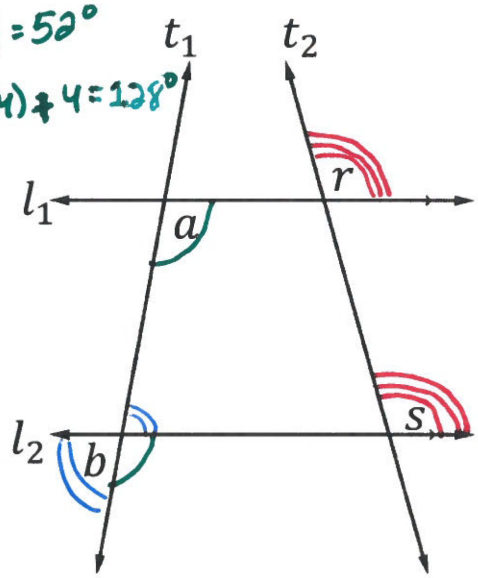
$+\ m\angle b = 31y + 4 = 31(4) + 4 = 128°$

$44y + 4$

$44y + 4 = 180$
$\quad -4 \quad\ -4$

$\frac{44y}{44} = \frac{176}{44}$

$y = 4$



$m\angle r = m\angle s$

$30x + 40 = 130x - 160$
$\quad -40 \qquad\quad -40$

$30x = 130x - 200$
$-130x \quad -130x$

$\frac{-100x}{-100} = \frac{-200}{-100}$

$x = 2$

$m\angle r = 30(2) + 40 = 100$

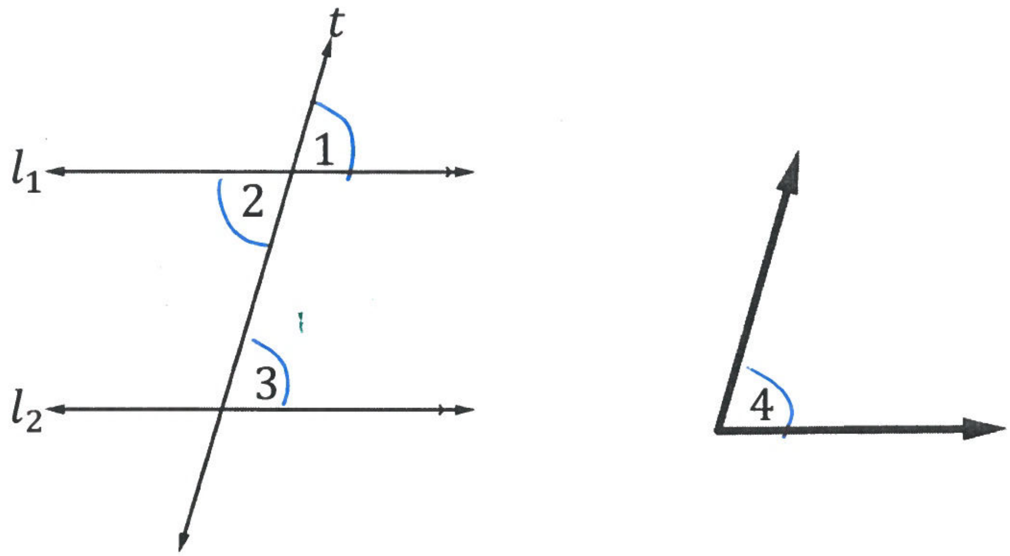
What are the values of $\angle a, \angle b, \angle r$, and $\angle s$?

$m\angle s = 130(2) - 160 = 100$

$\angle a =$ 52° $\angle b =$ 128°

$\angle r =$ 100° $\angle s =$ 100°

2. Consider the figure below.



Given: $l_1 \parallel l_2$; $\angle 2 \cong \angle 4$

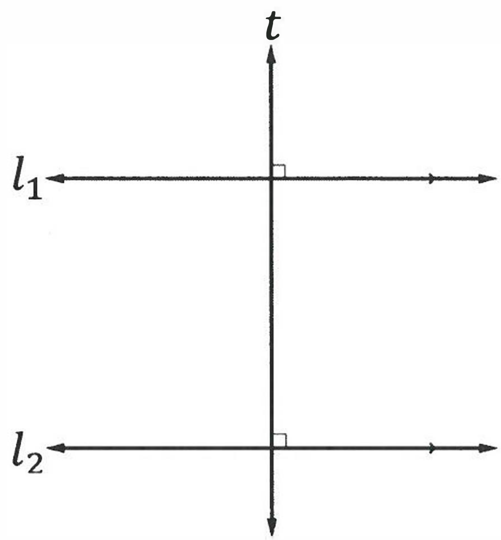
Prove: $\angle 1 \cong \angle 4$ and $\angle 4 \cong \angle 3$

Complete the following chart.

Statements	Reasons
1. $l_1 \parallel l_2$; $\angle 2 \cong \angle 4$	**1.** Given
2. $\angle 1 \cong \angle 2$	**2.** Vertical Angles Theorem
3. $\angle 1 \cong \angle 4$	**3.** Transitive Property
4. $\angle 1 \cong \angle 3$	**4.** Corresponding Angles Theorem
5. $\angle 4 \cong \angle 3$	**5.** Transitive Property of Congruence

Section 2 – Topic 8
Perpendicular Transversals

Consider the following figure of a transversal cutting parallel lines l_1 and l_2.



What observations can you make about the figure?

All angles are right angles.

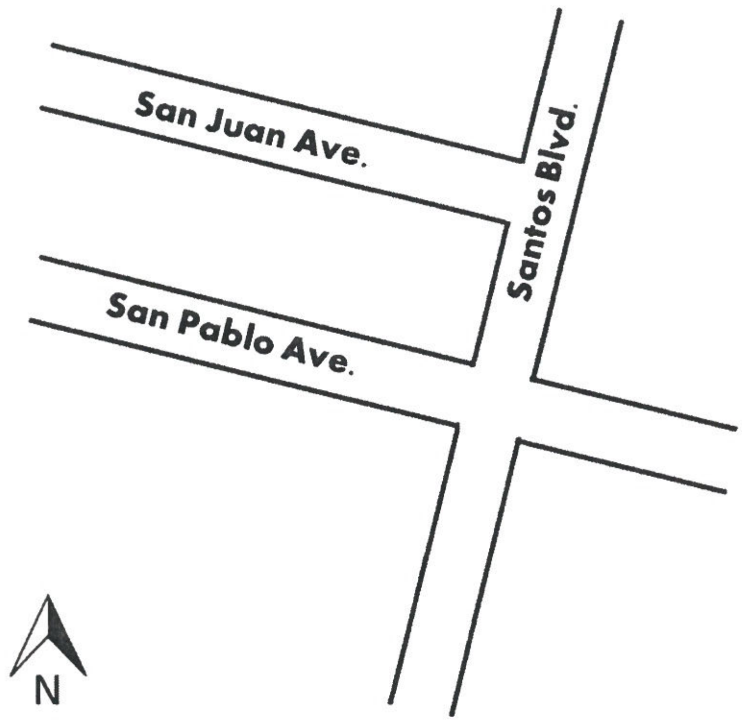
A transversal that cuts two parallel lines forming right angles is called a perpendicular transversal.

TAKE NOTE! *Postulates & Theorems*

Perpendicular Transversal Theorem:
In a plane, if a line is perpendicular to one of two parallel lines, then it is perpendicular to the other line also.

Perpendicular Transversal Theorem Corollary:
If two lines are both perpendicular to a transversal, then the lines are parallel.

Consider the figure below. San Pablo Ave. and Santos Blvd. are perpendicular to one another. San Juan Ave. was constructed later and is parallel to San Pablo Ave.

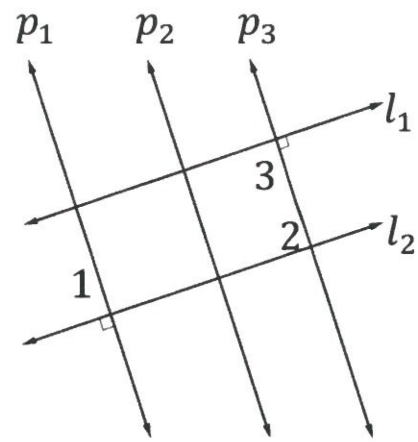


Using the Perpendicular Transversal Theorem, what can you conclude about the relationship between San Juan Ave. and Santos Blvd.?

They are perpendicular to each other.

Let's Practice!

1. Consider the following information.



Given: $p_1 \parallel p_2$, $p_2 \parallel p_3$, $l_2 \perp p_1$, and $l_1 \perp p_3$

Prove: $l_1 \parallel l_2$

Complete the following paragraph proof.

Because it is given that $p_1 \parallel p_2$ and $p_2 \parallel p_3$, then $p_1 \parallel p_3$ by

the __Transitive Property__.

This means that $\angle 1 \cong \angle$ __2__, because they are

corresponding angles.

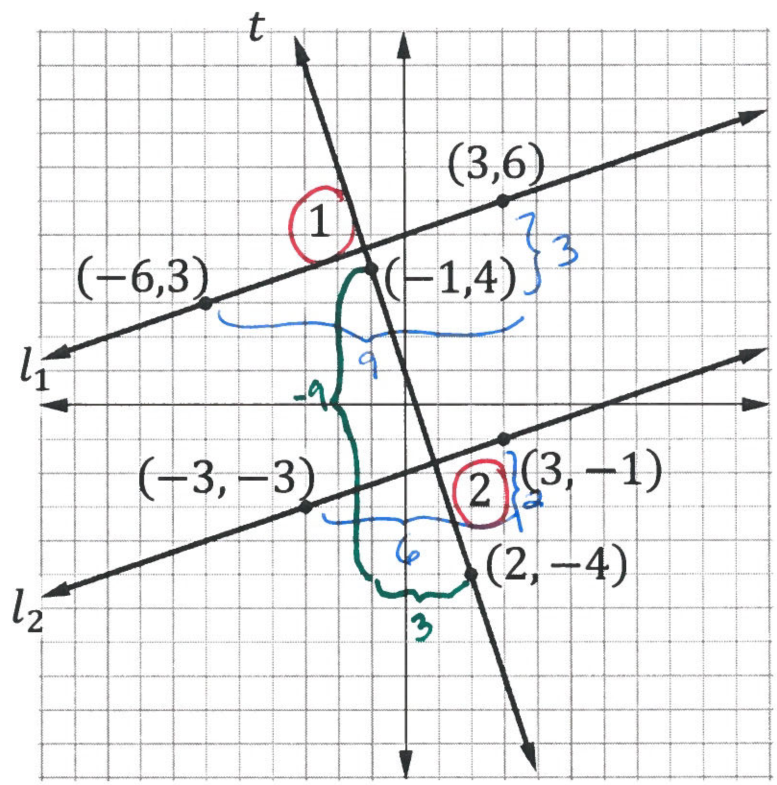
If $l_2 \perp p_1$, then $m\angle 1 = 90°$. Thus, $m\angle 2 =$ __90°__.

This means $p_3 \perp l_2$, based in the definition of perpendicular lines.

It is given that $l_1 \perp p_3$, so $l_1 \parallel l_2$ based on the corollary that states __If two lines are both perpendicular to a transversal, the lines are parallel.__.

Try It!

2. Consider the lines and the transversal drawn in the coordinate plane below.



a. Prove that $\angle 1 \cong \angle 2$. Justify your work.

Line ℓ_1 : $m = \frac{1}{3}$ $\quad \ell_1 \parallel \ell_2$ $\quad$ Definition of parallel lines

Line ℓ_2 : $m = \frac{1}{3}$ $\quad \angle 1 \cong \angle 2$ $\quad$ Alternate Exterior Angles Theorem

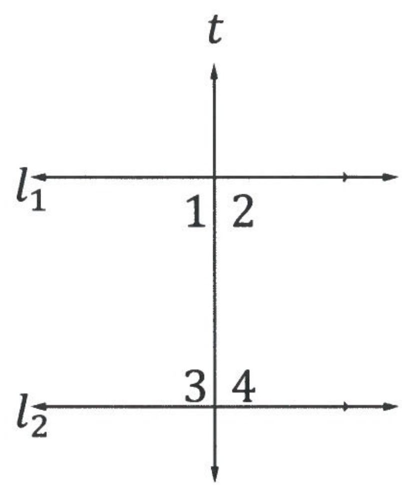
b. Prove that $m\angle 1 = m\angle 2 = 90°$. Justify your work.

Line ℓ_3 : $m = -3$ $\quad \ell_1 \perp t$ $\quad$ Definition of perpendicular lines.

All measurements are right angles $m\angle 1 = 90°$

BEAT THE TEST!

1. Consider the figure to the right, and correct the proof of the Perpendicular Transversal Theorem.



Given: $\angle 1 \cong \angle 4$ and $l_1 \perp t$ at $\angle 2$.
Prove: $l_2 \perp t$

Two of the reasons in the chart below do not correspond to the correct statement. Circle those two reasons.

Statements	Reasons
1. $\angle 1 \cong \angle 4$; $l_1 \perp t$ at $\angle 2$	**1.** Given
2. $l_1 \parallel l_2$	**2.** Consecutive Angles Theorem
3. $\angle 2$ is a right angle	**3.** Definition of perpendicular lines
4. $m\angle 2 = 90°$	**4.** Definition of right angle
5. $m\angle 2 + m\angle 4 = 180°$	**5.** Converse of Alternate Interior Angles Theorem
6. $90° + m\angle 4 = 180°$	**6.** Substitution property
7. $m\angle 4 = 90°$	**7.** Subtraction property of equality
8. $l_2 \perp t$	**8.** Definition of Perpendicular Lines

Section 2 – Topic 9
Proving Angle Relationships in Transversals and Parallel Lines

Consider a transversal passing through two parallel lines.

How do you know that a pair of alternate interior angles or a pair of corresponding angles are congruent under this scenario? The Alternate Interpers, Angles Theorem and the Corresponding Angles Theorem

How do you know that same-side consecutive angles are supplementary under this scenario? The Same-Side/ Consecutive Angles Theorem

How do you prove your answers?

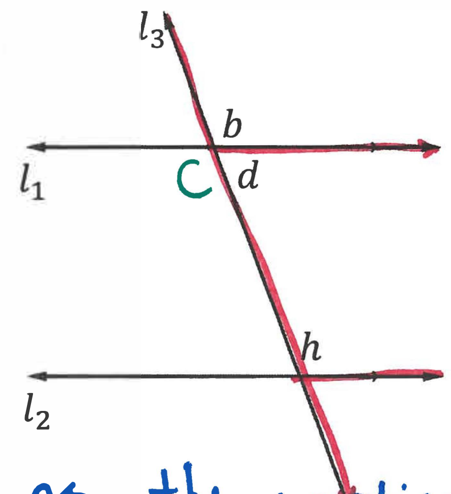
Two-colunn proof, photograph proof, flow chart proof, and Contruction

Let's Practice!

1. Consider the lines l_1, l_2 and l_3 in the diagram at the right.

   Given: $l_1 \parallel l_2$
   Prove: $\angle b \cong \angle h$

   Write a paragraph proof.



Let label angle C as the vertical angle of b.

Since $l_1 \parallel l_2$, $\angle c \cong \angle h$ because of the Alternate Interior Angles Theorm. We also know know that $\angle c \cong \angle b$, because of the Vertical Angles Theorum. Finally, we can state that $\angle b \cong \angle h$ because of the Transitive Property of Congruence.

2. Reconsider the diagram and proof of exercise #1. Determine how the use of a rigid transformation is a good alternative to prove that $\angle b \cong \angle h$.

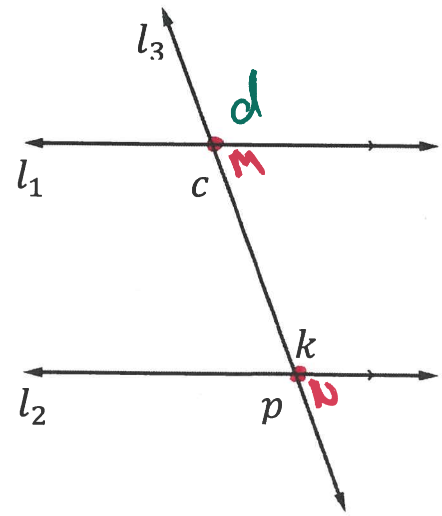
Translate $\angle b$ down l_3 so $\angle b$ is mapped onto $\angle h$.
Translation preserves size, angles, and direction ; so $\angle b \cong \angle h$.

Try It!

3. Consider the lines l_1, l_2 and l_3 in the diagram to the right.

Given: $l_1 \parallel l_2$
Prove: $\angle c \cong \angle k$

Complete the following proof.



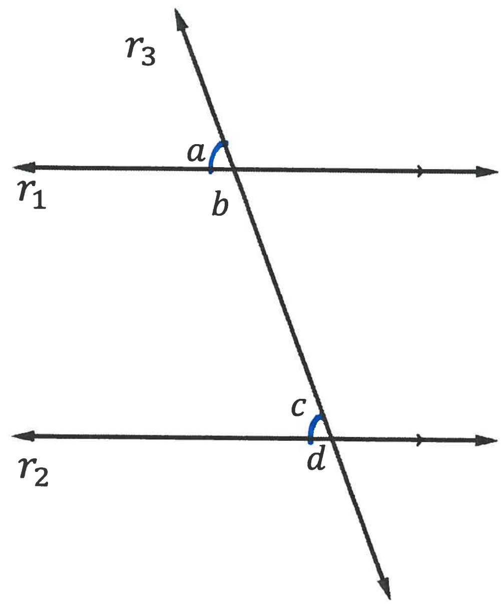
Statements	**Reasons**
1. $l_1 \parallel l_2$	**1.** Given
2. $\angle c \cong \angle p$	**2.** Corresponding Angles Theorem
3. $\angle k \cong \angle p$	**3.** Vertical Angles Theorem
4. $\angle c \cong \angle k$	**4.** Transitive Property of Congruence

4. Determine how the use of a rigid transformation is good for an alternative to prove that $\angle c \cong \angle k$.

Let use d to lable the angle that is vertical to c. Let label the intesections of l_1 and l_3, and l_2 and l_3 as M and N, respectively. If we rotate $\angle C$ 180° it will match $\angle d$ if center of rotation is M. Now, we can translate $\angle C$ down l_3 so $\angle C$ is mapped onto $\angle k$.

BEAT THE TEST!

1. Consider the diagram below.



Given: $r_1 \parallel r_2$
Prove: $m\angle b + m\angle c = 180°$

Complete the following paragraph proof by circling the correct answer in each shaded part.

Since $r_1 \parallel r_2$ and r_3 is a transversal, $\angle a$ and $\angle b$ form a linear pair, same as $\angle c$ and $\angle d$ by definition. Therefore, each pair of angles ($\angle a$ and $\angle b$, and $\angle c$ and $\angle d$) are supplementary according to the Linear Pair Postulate. So, $m\angle a + m\angle b = 180°$ and $m\angle c + m\angle d = 180°$ by definition. Since $\angle a \cong \angle c$ according to the Alternate Interior | Corresponding | Vertical Angles Theorem, $m\angle a = m\angle c$ by definition of congruency. Hence, $m\angle b + m\angle c = 180°$ according to reflexive property | substitution | transitive property of congruence. This proves the Same-side Consecutive Angles Theorem.

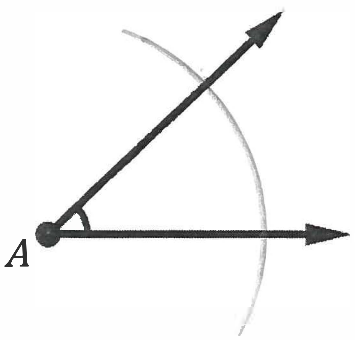
Section 2 – Topic 10
Copying Angles and Constructing Angle Bisectors

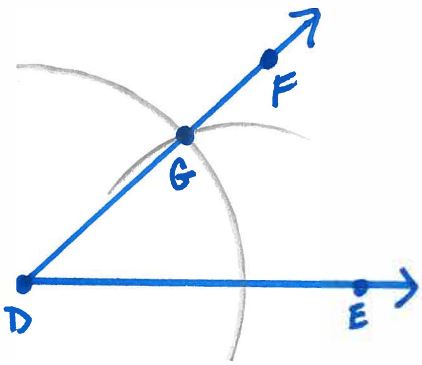
What information or tools do we need to construct an angle?

Compass and straightedge (perhaps a protractor)

Now, an angle already exists and we want to construct another angle that is exactly the same, then we are Copying that angle.

Let's consider $\angle A$. Construct $\angle FDE$ to be a copy of $\angle A$.





Step 1. Draw a ray that will become one of the two rays of the new angle. Label the ray $\overrightarrow{DE}$.

Step 2. Place your compass point at the vertex of $\angle A$. Create an arc that intersects both rays of $\angle A$.

Step 3. Without changing your compass setting, create an arc from point D that intersects $\overrightarrow{DE}$. Be sure to make a large arc.

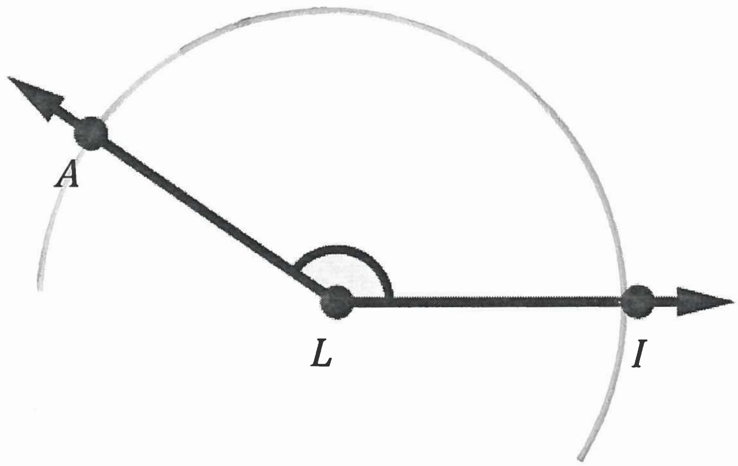
Step 4. On $\angle A$, set your compass point on the intersection of the arc and ray and the pencil on the other intersection of the arc and second ray. Lock your compass.

Step 5. Place the point of the compass on the intersection of the arc and $\overrightarrow{DE}$. Mark an arc through the large arc created in step 3. Label the point of intersection of the two arc point G.

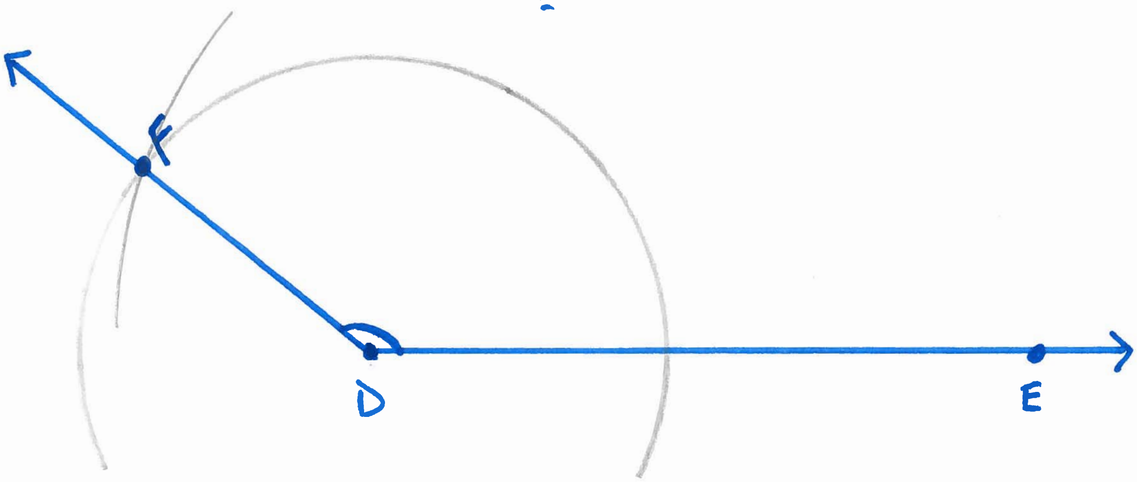
Step 6. Construct $\overrightarrow{DG}$.

Let's Practice!

1. Consider $\angle ALI$.



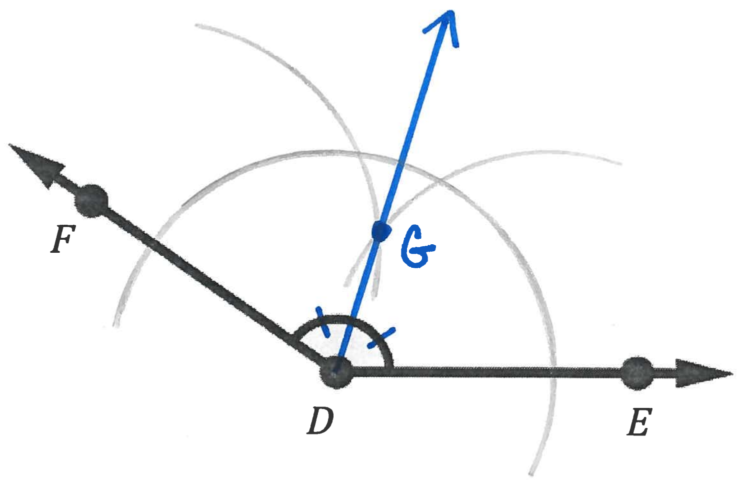
Part A: Construct $\angle FDE$ to be a copy of $\angle ALI$.



Part B: Suppose that your teacher asks you to construct an angle bisector to $\angle FDE$. How would you do it?

Find a way to create a point above the angle that equidistant from the intersection of the first arc with $\overrightarrow{DF}$ and $\overrightarrow{DE}$.

In order to bisect an angle, follow these steps and perform the construction of the angle bisector in $\angle FDE$ below.



Step 1. Place the point of the compass on the angle's vertex.

Step 2. Without changing the width of the compass, draw an arc across each ray.

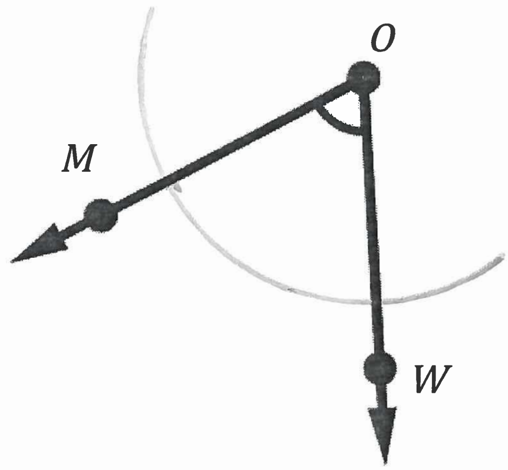
Step 3. Place the point of the compass on the intersection of the arc and the ray draw an arc in the interior of the angle.

Step 4. Without changing the compass setting, repeat step 3 for the other angle so that the two arcs intersect interior of the angle. Label the intersection G.

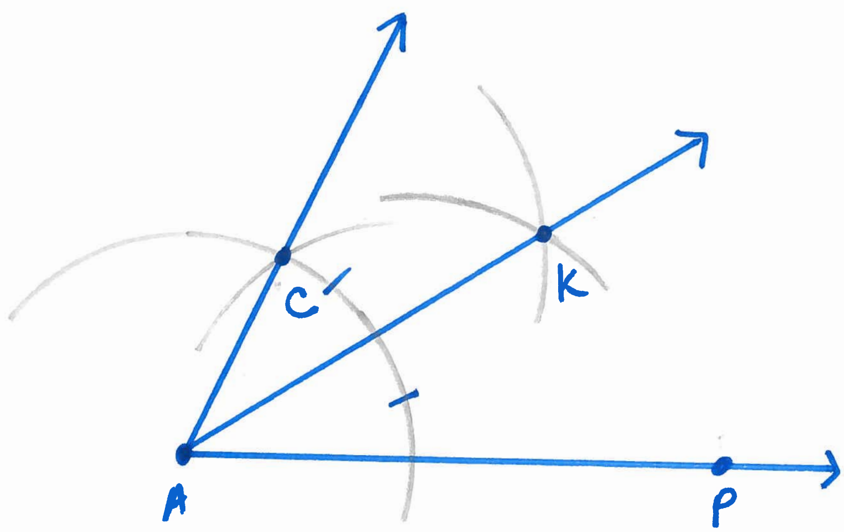
Step 5. Using a straightedge, construct a ray from the vertex D, through the point where the arcs intersect, G.

Try It!

2. Consider $\angle MOW$.

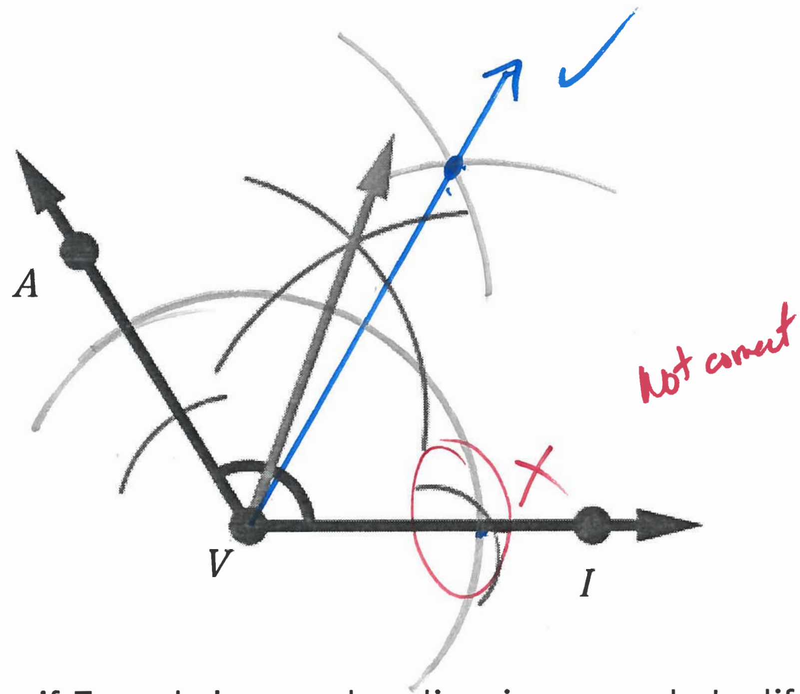


Construct $\angle CAP$ to be a copy of $\angle MOW$ and bisect $\angle CAP$ with $\overrightarrow{AK}$.



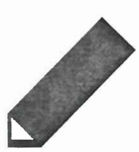
BEAT THE TEST!

1. Ernesto bisected $\angle AVI$ and his construction is shown below.



Determine if Ernesto's construction is correct. Justify your answer.

- The arc on $\overrightarrow{VI}$ does not look correct. Please check!
- The point where the two arcs above the angle AVI intersect is not equidistant to the arcs on the rays, precisely the point where the arcs intersect the rays.



Test Yourself! Practice Tool

Great job! You have reached the end of this section. Now it's time to try the "Test Yourself! Practice Tool," where you can practice all the skills and concepts you learned in this section. Log in to Math Nation and try out the "Test Yourself! Practice Tool" so you can see how well you know these topics!

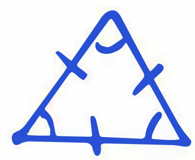
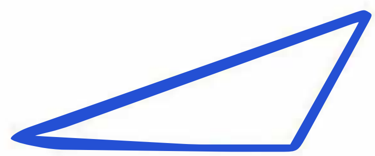
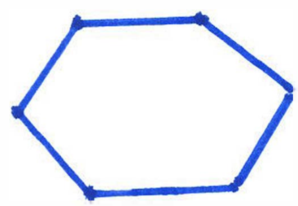
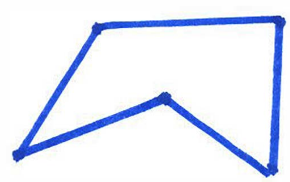
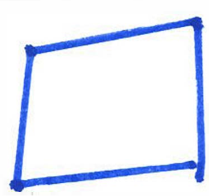
Section 2 – Topic 11
Introduction to Polygons

The word polygon can be split into two parts:

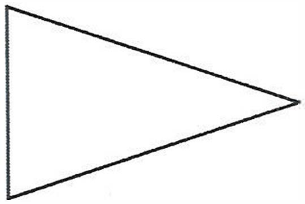
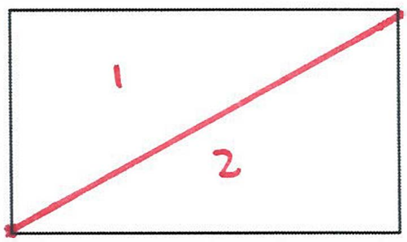
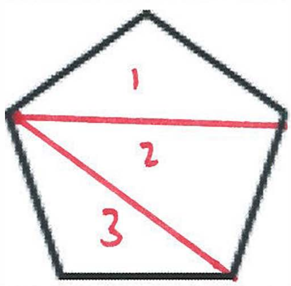
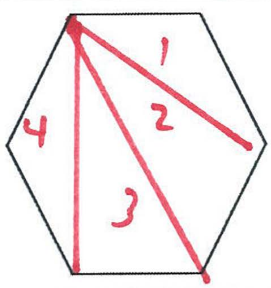
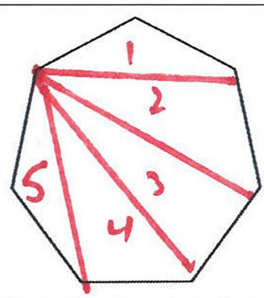
- "poly-" means many
- "gon" means sides

Polygons are simple, closed, and have sides that are segments.

Draw a representation for each of the polygons below.

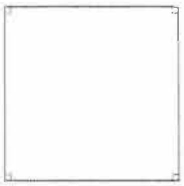
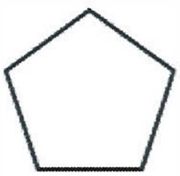
Name	Definition	Representation
Regular	All angles and sides of this polygon are congruent.	
Irregular	All angles and sides of this polygon are not congruent.	
Convex	This polygon has no angles pointing inwards. That is, no interior angles can be greater than 180°.	
Concave	This polygon has an interior angle greater than 180°.	
Simple	This polygon has one boundary and doesn't cross over itself.	

Complete the table by using your knowledge of triangles to find the sum of the interior angles of each polygon.

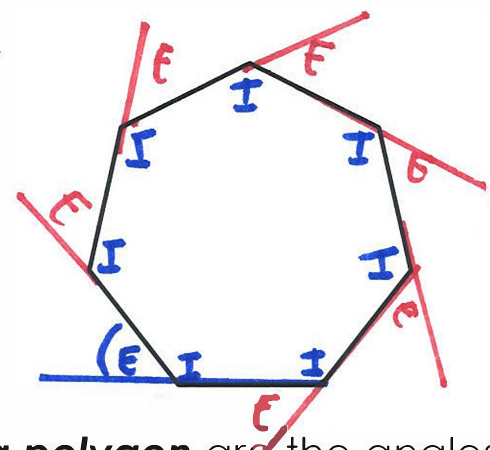
Polygon	Number of sides	Sum of interior angles
	3	180°
	4	$2(180) = 360°$
	5	$3(180) = 540°$
	6	$4(180) = 720°$
	7	$5(180) = 900°$
	n	$(n-2)(180)°$

Try it!

1. Classify each figure as regular, concave, and/or convex by marking the appropriate box. Name each type of polygon represented by filling in each blank provided.

Figure	Regular	Concave	Convex	Name the Polygon
	☑	☐	☑	Quadrilateral
	☐	☑	☐	Decagon
	☑	☐	☑	Pentagon

Consider the polygon.



The ***interior angles of a polygon*** are the angles on the inside of the polygon formed by each pair of adjacent sides.

Use I to label the interior angles of the polygon above.

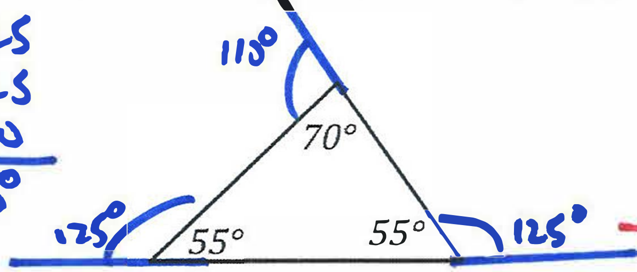
An ***exterior angle of a polygon*** is an angle that forms a linear pair with one of the interior angles of the polygon.

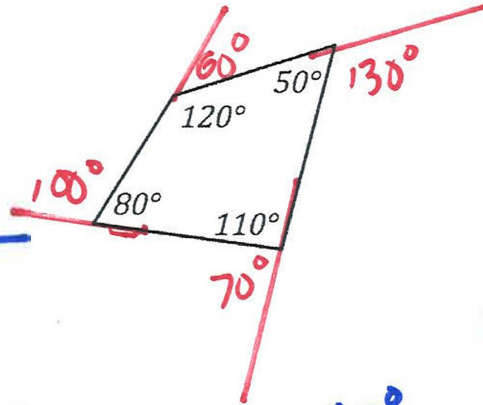
Use E to label the exterior angels of the polygon above.

Let's Practice!

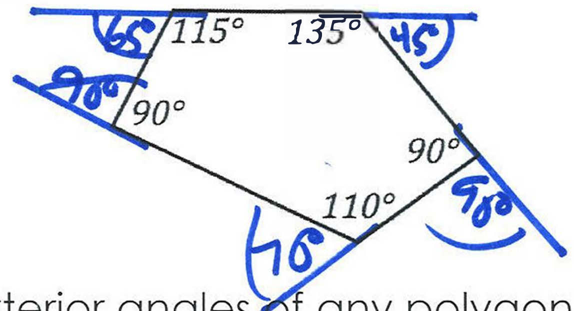
2. Consider each of the following polygons. Find the sum of the exterior angles in each polygon below.

$$\begin{array}{r} 125 \\ 125 \\ +\,110 \\ \hline 360^\circ \end{array}$$





$$\begin{array}{r} 100 \\ 60 \\ 130 \\ +\ 70 \\ \hline 360^\circ \end{array}$$



$$\begin{array}{r} 65^\circ \\ 45^\circ \\ 90^\circ \\ 90^\circ \\ +\ 70^\circ \\ \hline 360^\circ \end{array}$$

The sum of the exterior angles of any polygon equals 360°.

Try it!

3. A convex pentagon has exterior angles with measures 77°, 66°, 82°, and 62°.
   a. What is the measure of the exterior angle of the pentagon at the fifth vertex?

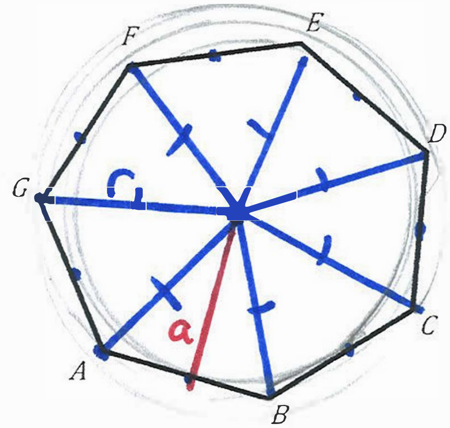
   $360^\circ - (77^\circ + 66^\circ + 82^\circ + 62^\circ)$

   $360^\circ - 287^\circ = 73^\circ$

   b. Classify the pentagon as regular or irregular. Justify your answer.

   Since the Exterior angles are not congruent, we conclude the polygon is irregular.

Consider the following regular heptagon.



The center of the heptagon is marked.

- The ***circumcenter*** is the point that is equidistant from each vertex.

Draw a circle outside the heptagon that only touches the vertices of the heptagon.

- The "outside" circle is called a circumcircle, and it connects all the vertices of the polygon.

Draw a circle inside that only touches each side of the heptagon at its midpoint.

- The "inside" circle is called an incircle, and it connects all the midpoints of the sides of the polygon.

Draw a line from the center of the heptagon to one of its vertices.

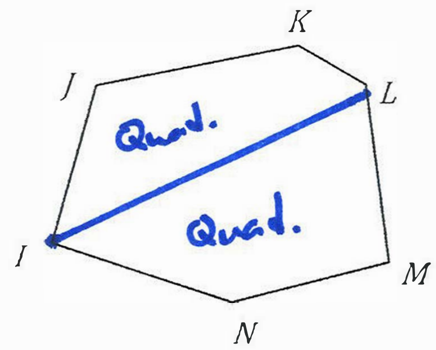
- This is called the radius of the polygon, which is also the radius of the circumcircle.

Draw all the radii of the heptagon. It should result in seven isosceles triangles.

- The height of each isosceles triangle is also called the apothem of the polygon and the radius of the incircle.

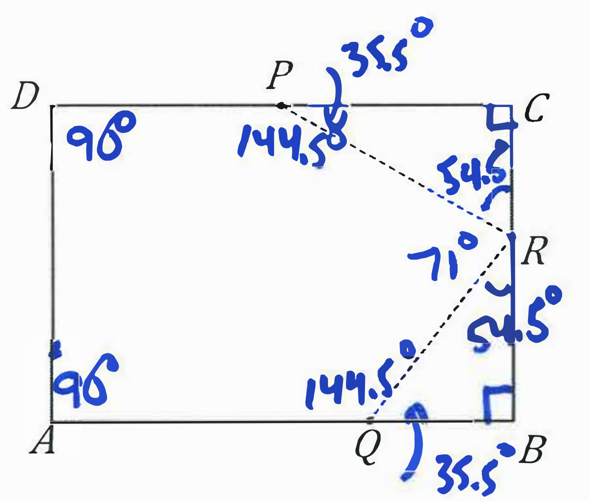
BEAT THE TEST!

1. Consider the irregular hexagon below.



Provide one way to break up the irregular polygon above using smaller polygons. Identify each type of smaller polygon you form.

2. Rectangle $ABCD$ was cut to create pentagon $AQRPD$ in the figure below.



$180-71=109°$

$\frac{109}{2}=54.5°$

If $m\angle PRQ = 71°$ and $m\angle PRC = m\angle QRB$, verify the sum of the interior angles of pentagon $AQRPD$ using two different methods. Justify your answers.

$(n-2)180°$

$(5-2)180° = 540°$

$90+90+144.5+144.5+71$

$= 540°$

Read the following statement. What can you logically conclude?

If $m\angle A$ is less than $90°$, then $\angle A$ is an acute angle.

$$m\angle A = 85° .$$

Since $m\angle A < 90°$, $\angle A$ is an acute angle.

Deductive reasoning is a type of reasoning using given and previously known facts to reach a logical conclusion.

In this course, we will use deductive reasoning to prove statements. There are three different types of proofs:

Type of Proof	Definition
two-column proof	uses a table and explicitly places the statements in the first column and the reasoning in the second column
paragraph proof	the statements and their reasoning are written together in a logical order in paragraph form
flow chart proof	a concept map where statements are placed in the boxes and the reason for each statement are placed under the box

Let's Practice!

1. Complete the two-column proof to prove that $x = 5$.

Given: $LM = 3x + 1$
$MN = x + 2$
$LN = 23$
Prove: $x = 5$

Statements	Reasons
1. $LM = 3x + 1$ $MN = x + 2$ $LN = 23$	1. Given
2. $3x+1 + x+2 = 23$	2. Segment Addition Postulate
3. $4x + 3 = 23$	3. Equivalent Equation
4. $4x+3-3 = 23-3$	4. Addition Property of Equality
5. $4x = 20$	5.
6. $\left(\frac{1}{4}\right)4x = 20\left(\frac{1}{4}\right)$	6. Multiplication Property of Equality
7. $x = 5$	7. Equivalent Equation.

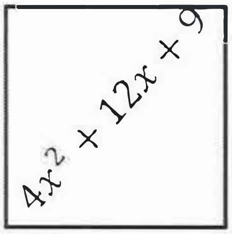
What will the *first* row of a two-column proof always be?

The given statement(s).

What will the *last* row of a two-column proof always be?

The statement we are trying to prove.

2. The given figure is a square. The expression represents the area of the square. Use a paragraph proof to show that the length of one side of the square is $(2x + 3)$.



Given: Figure is a square. Area of the square is $4x^2+12x+9$.

Prove: One side of the square is $2x+3$.

We are given the area of a square is represented by the expression $4x^2+12x+9$. By definition, the area of a square is the length of a side squared. We can find an equivalent expression that $4x^2+12x+9=(2x+3)(2x+3)$. Therefore, the length of one side of the square is $(2x+3)$.

3. Use the word bank to prove the conditional using a flow chart proof.

If $\frac{3x}{x+5} = 2$, then $x = 10$.

~~Given~~	~~Subtraction Property~~	$\frac{3x}{x+5} = 2$
$3x = 2(x + 5)$		~~Distributive Property~~
	$x = 10$	$3x = 2x + 10$
~~Multiplication Property~~		

START → $\frac{3x}{x+5} = 2$ → $3x = 2(x+5)$ → $3x = 2x+10$ → $x = 10$ → END

Given | Multiplication Property | Distributive Property | Subtraction Property

Try It!

$\{1, 2, 3, 4, 5, ...\}$

4. When a natural number is added to three and the sum is divided by two, the quotient will be an even number.

Which of the following is a counterexample to the statement above?

Ⓐ $\frac{13+3}{2} = 8$, which is an even number. ✓ True

Ⓑ $\frac{12}{2} + 3 = 9$, which is not an even number. X

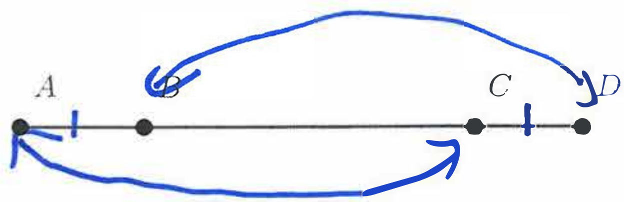
● $\frac{3+4}{2} = \frac{7}{2}$, which is not an even number. ✓ True

Ⓓ The statement is correct. There is no counterexample. X

BEAT THE TEST!

1. Consider the diagram below and finish the two-column proof to show $AC = BD$.

Given: $AB = CD$
Prove: $AC = BD$



Statements	Reasons
1. $AB = CD$	1. Given
2. BC = BC	2. Reflexive Property
3. $AB + BC = BC + CD$	3. Addition Property of equality
4. $AB + BC = AC$ $BC + CD = BD$	4. Segment addition postulate.
5. AC = BD	5. Substitution

Section 2 – Topic 12
Angles of Polygons

In the previous video, you learned the formula to find the sum of the angles of a polygon.

$$S = 180(n-2)$$

How can you use the sum of interior angles formula to find the number of sides of a polygon?

$$\frac{180(n-2)}{180} = \frac{S}{180} \qquad n-2 = \frac{S}{180}$$

$$n = \frac{S}{180} + 2$$

How can you use the sum of interior angles formula to find the measure of one angle of a regular polygon?

$$\frac{180(n-2)}{n}$$

Can the same process be used to find the measure of one angle of an irregular polygon? Explain your reasoning.

No. All the angles are not the same.

Let's Practice!

1. What are the measures of each interior angle and each exterior angle of regular hexagon $MARLON$?

$n=6$

Each interior

$$\frac{180(n-2)}{n} = \frac{180(6-2)}{6} = 120^\circ$$

Each exterior

$$\frac{360^\circ}{6} = 60^\circ$$

2. The sum of the interior angles of a regular polygon is 1080°.

   a. Classify the polygon by the number of sides.

$$n = \frac{S}{180} + 2$$

$$n = \frac{1080}{180} + 2$$

$n = 6 + 2$

$n = 8$

Octagon

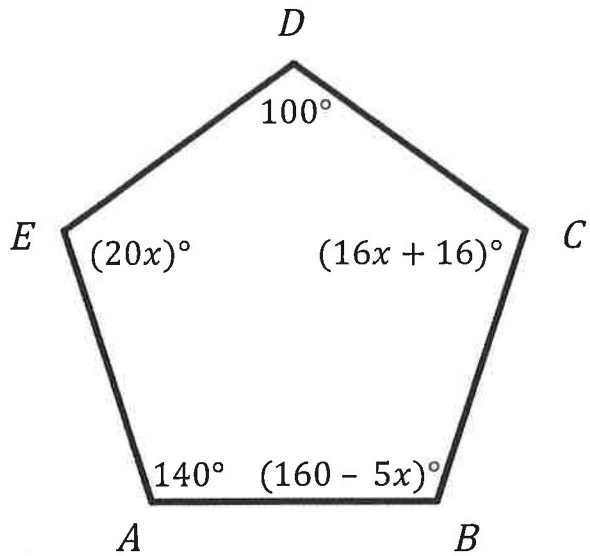
   b. What is the measure of one interior angle of the polygon?

$$\frac{180(n-2)}{n} = \frac{1080}{8} = 135^\circ$$

   c. What is the measure of one exterior angle of the polygon?

$$\frac{360^\circ}{n} = \frac{360^\circ}{8} = 45^\circ$$

3. Consider pentagon $ABCDE$.



$n = 5$

$180(n-2)$

$= 180(5-2)$

$= 180(3) = 540°$

a. Find the value of x.

$140 + 20x + 100 + 16x + 16 + 160 - 5x = 540$

$416 + 31x = 540$

$31x = 124$

$x = 4$

b. Find the value of the following angles: $\angle A$, $\angle B$, $\angle C$, $\angle D$, and $\angle E$.

$m\angle A = 140°$

$m\angle B = 160 - 5x = 160 - 5(4) = 140°$

$m\angle C = 16x + 16 = 16(4) + 16 = 80°$

$m\angle D = 100°$

$m\angle E = 20x = 20(4) = 80°$

c. Find the value of each exterior angle.

$m\angle A = 140°$

$m\angle B = 140°$

$m\angle C = 80°$

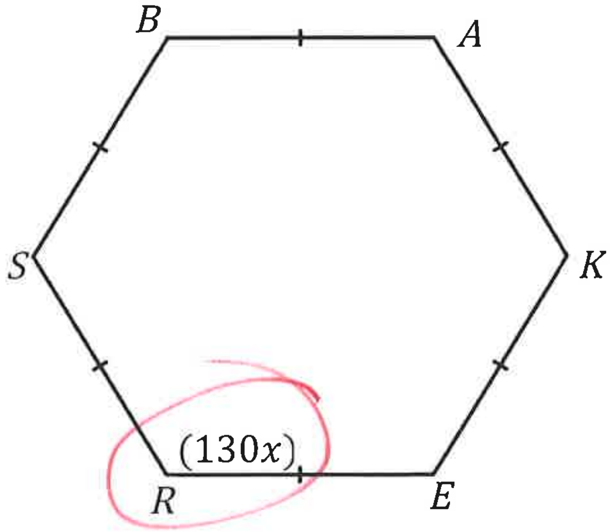
$m\angle D = 100°$

$m\angle E = 80°$

$\rightarrow$ Exterior angles $\left\{ \begin{array}{l} 40° \\ 40° \\ 100° \\ 80° \\ 100° \end{array} \right.$

Try It!

4. Consider the regular hexagon below.



$n=6$

✓ Find the value of x and determine the value of each interior and exterior angle.

$\frac{180(n-2)}{n} = \frac{180(6-2)}{6} = \frac{180(4)}{6} = \frac{720}{6} = 120°$ Interior ∡

$\frac{360}{6} = 60°$ Exterior ∡

$130x = 120$ $x = \frac{120}{130}$ $x = \frac{12}{13}$

5. If the measure of an exterior angle of a regular polygon is 24°, how many sides does the polygon have?

$\frac{360°}{n} = 24°$ $n = \frac{360}{24} = 15$

$360° = 24°n$ 15 sides

6. Given a regular decagon and a regular dodecagon, which one has a greater exterior angle? By how much is the angle greater?

$n=10$ $n=12$

$\frac{360}{10} = 36°$ $\frac{360}{12} = 30°$

Decagon by 6°

BEAT THE TEST!

1. A teacher showed the following exit ticket on the projector.

> 1. What is the sum of the interior angle measures of a regular 24-gon?
> 2. Pentagon ABCDE has interior angles that measure 60° and 160° and another pair of interior angles that measure 130° each. What is the measure of an interior angle at the fifth vertex?

A student completed the following exit ticket.

1) $\text{Sum} = \dfrac{(n-2)180}{\cancel{n}}$

$= \dfrac{(24-2)180}{\cancel{24}}$

$= \dfrac{(22)180}{\cancel{24}} = 3960°$

$\cancel{= 165}$

2) $130 + 130 + 60 + 160 + x = (5-2)180$

$\cancel{350 + x = 540}$

$\cancel{x = 190}$

$450 + x = 540$

$x = 60°$

Which of the following statements is true?

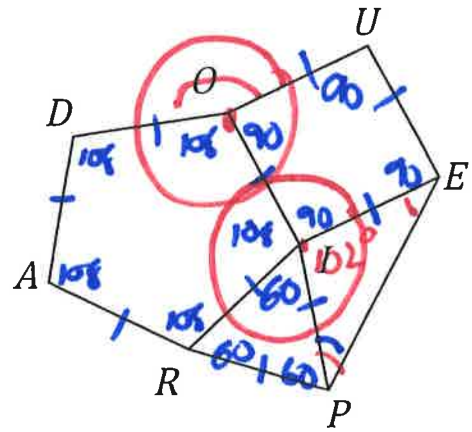
Ⓐ Both answers are correct.

Ⓑ Answer #1 is incorrect. The student found the individual angle, not the sum of the angles. The answer should be 3960°. Answer #2 is correct.

Ⓒ Answer #1 is correct. Answer #2 is incorrect. There are two angles measuring 130°, but only one was counted in the sum. The answer should be 60°.

Ⓓ Both answers are incorrect. In #1 the student found the individual angle, not the sum of the angles. The answer is 3960°. In #2 there are two angles measuring 130°, but only one was counted in the sum. The answer should be 60°.

2. Consider the figure below.



$\frac{540}{5} = 108$

$360 - 108 - 90 - 60$

$= 102°$

$DARIO$ is a regular pentagon, RIP is an equilateral triangle, and $EIOU$ is a square.

Part A: What is the measure of $\angle IPE$?

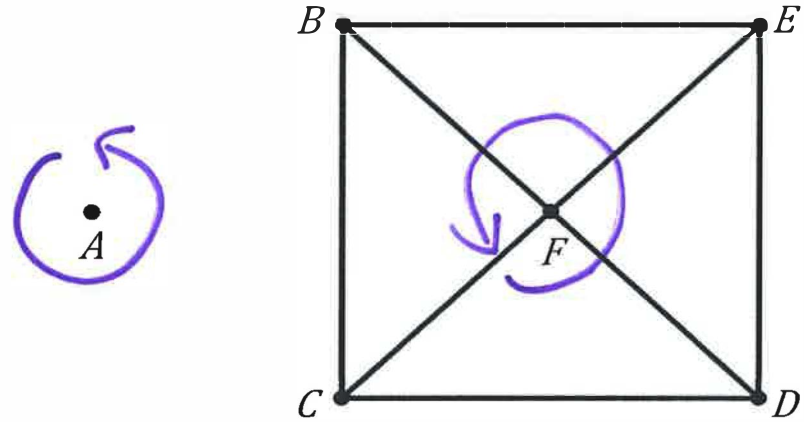
$m\angle IPE = \frac{180° - 102°}{2} = \frac{78°}{2} = 39°$

Part B: Find $m\angle DOU$.

$m\angle DOU = 360 - 108 - 90 = 162°$

Section 2 – Topic 13
Angles of Other Polygons

Use the following diagram, where point A and square $BCDE$ with center at F are shown, to answer the questions below.



What is the angle measure surrounding point A? 360°

What are the angle measures surrounding point F? 360°

What is the difference between points A and F?

Point A has one angle, and F has 4 angles

Some important facts about the angles of a polygon:

- The ***center*** of a polygon is the point equidistant from every vertex.
- The ***central angle*** of a polygon is the angle made at the center of the polygon by any two adjacent vertices of the polygon.
- The sum of the central angles of a polygon is 360° (a full circle).
- The measure of the central angle of a regular polygon is 360° divided by the number of sides.

The base angles of each isosceles triangle in a regular polygon can be calculated in two ways.

- Base angles of an isosceles triangle are equal. Therefore, each base angle can be calculated by $\frac{180-c}{2}$, where c is a central angle.

- The radius of a polygon bisects the angle at the vertex and each interior angle of a regular polygon is $\frac{180(n-2)}{n}$, where n is the number of sides of the regular polygon.

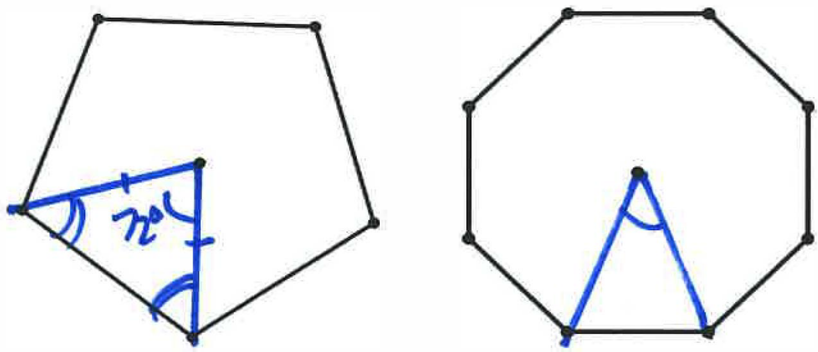
Pentagon

$$\frac{180(5-2)}{5} = 108$$

bisected $\frac{108}{2} = 54°$

Let's Practice!

1. Consider the following diagram of the regular polygons.



a. Draw a central angle in each of the above polygons and calculate the measure of a central angle in each polygon.

Pentagon $\frac{360°}{5} = 72°$

Octagon $= \frac{360°}{8} = 45°$

b. What are the measures of the base angles of each isosceles triangle in the pentagon?

$180 - 72 = 108$

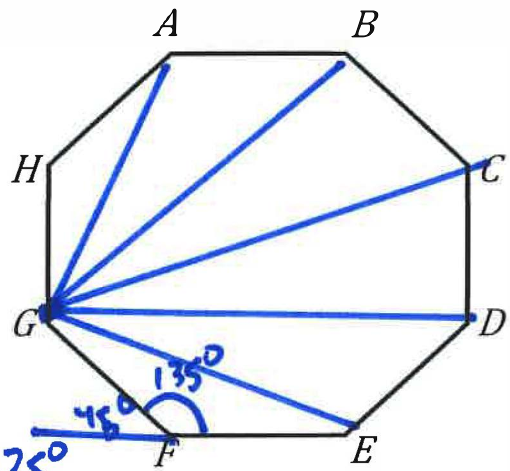
$\frac{108}{2} = 54°$

c. What are the measures of the base angles of each isosceles triangle in the octagon?

$180 - 45 = 135$

$\frac{135}{2} = 67.5°$

2. Consider the following regular octagon, and use it to complete the questions below.



$\frac{180(n-2)}{n}$

$\frac{180(8-2)}{8} = 135^\circ$

a. Prove that the sum of all exterior angles is $360°$ in a regular octagon.

$45^\circ(8) = 360^\circ$

b. Prove that the sum of all interior angles at each vertex is $180(n-2)$ in a regular octagon.

$180(6) = 1080^\circ$

$180(8-2)$

$180(n-2)$

3. A student claims that the sum of the measures of the exterior angles of a hendecagon is greater than the sum of the measures of the exterior angles of a nonagon. The student justifies this claim by saying that a hendecagon has two more sides than the nonagon. 11 9

Describe and correct the student's error.

All exterior angles of any polygon add to 360°.

4. Determine if an irregular polygon has a central angle. Justify your answer.

No. Vertex will not fall on a circumcircle for an irregular polygon.

5. Does an irregular polygon have exterior angles? If so, how do we calculate the exterior angles? Justify your answer.

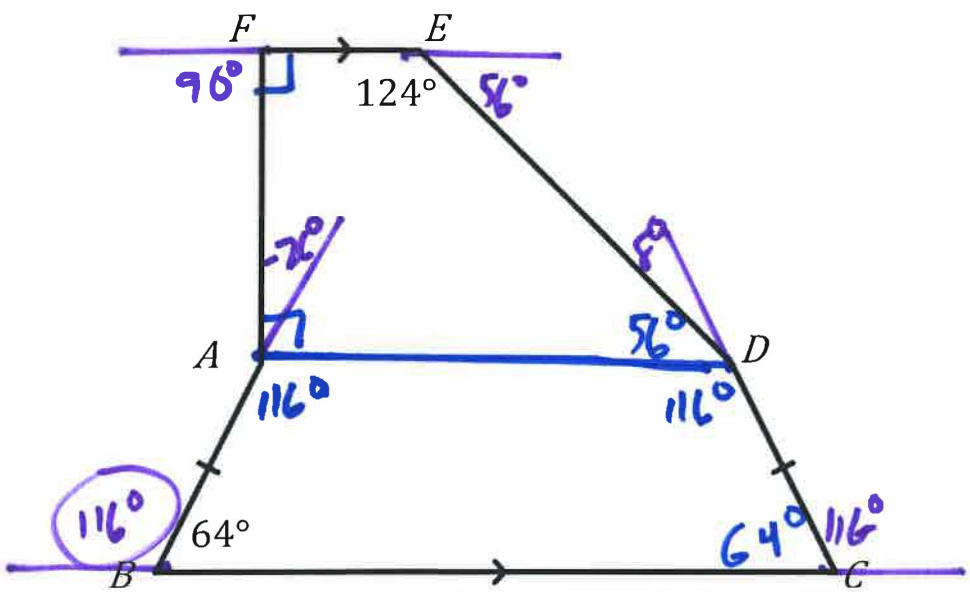
Yes. Separate into smaller polygons to calculate.

STUDY EDGE TIP

Irregular polygons do not have a center, and they do not have a central angle; however, they do have interior and exterior angles.

Try It!

6. Consider the following irregular hexagon and answer the questions below it.



a. If $\overline{AF} \perp \overline{EF}$ and $\overline{AF} \perp \overline{AD}$, find the measure of each interior angle of the irregular polygon above.

$m\angle A: 206°$ $m\angle C: 64°$ $m\angle E: 124°$

$m\angle B: 64°$ $m\angle D: 172°$ $m\angle F: 90°$

b. Does the same exterior angles rule for regular polygons apply to irregular polygons? Justify your answer. $90° + 56° + 8° + 116° + 116° + -26° = 360°$

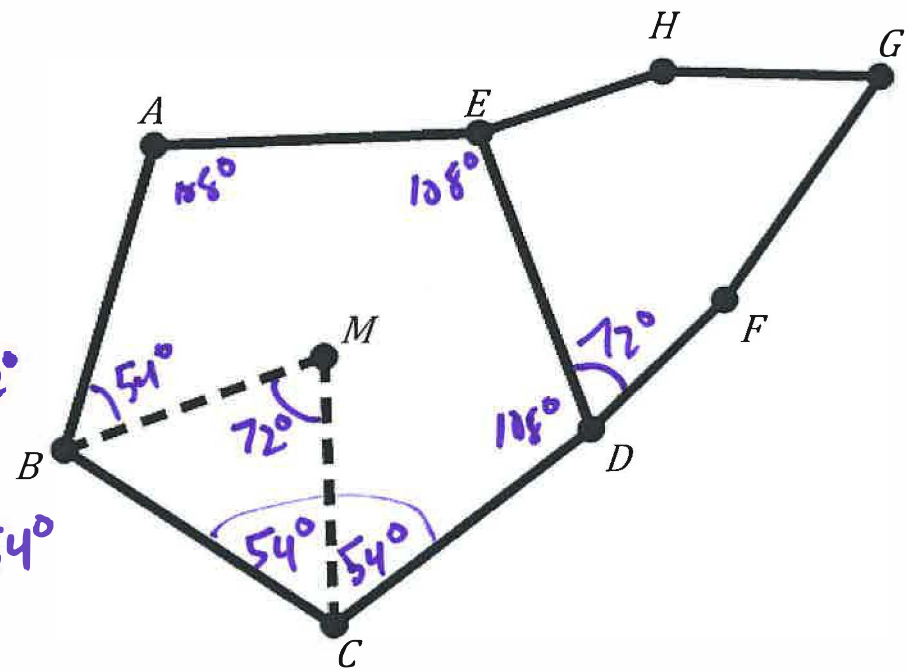
Yes!

BEAT THE TEST!

1. Consider the following diagram where the regular polygon $ABCDE$ has center at M, polygon $DEHGF$ is irregular, and point D is on $\overline{CF}$.

$\frac{360}{5} = 72°$

$\frac{180-72}{2} = 54°$



Which of the following statements are correct? Select all that apply.

- [x] $m\angle BMC = m\angle EDF$
- [ ] $m\angle EDC = 72°$
- [ ] The sum of the exterior angles of $ABCDE$ ~~is less~~ = than the sum of the exterior angles of $DEHGF$.
- [x] The sum of the interior angles of polygon $ABCDE$ 540 with the sum of the exterior angles of polygon $DEHGF$ 360 equals 900°.
- [ ] $m\angle ABM = \cancel{m\angle BMC} = m\angle DCM$

Test Yourself! Practice Tool	Great job! You have reached the end of this section. Now it's time to try the "Test Yourself! Practice Tool," where you can practice all the skills and concepts you learned in this section. Log in to Math Nation and try out the "Test Yourself! Practice Tool" so you can see how well you know these topics!