<u>**Basics of Geometry – Part 1**</u>

What is **geometry**? Visual spatial branch of math. Its concerned with measurements of length, area, volume, perimeters, circumferences, and so on.

Geometry means "earth measurement," and it involves the properties of points, lines, planes and figures.

What concepts do you think belong in this branch of mathematics?

angles, shapes, dimensions, proofs, points, lines, planes, figures...

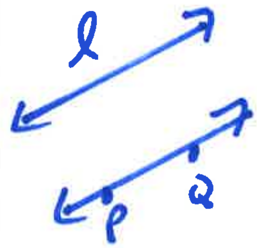
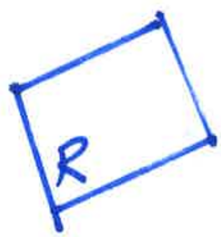
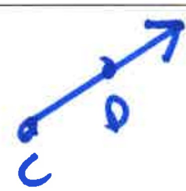
Why does geometry matter? When is geometry used in the real world?

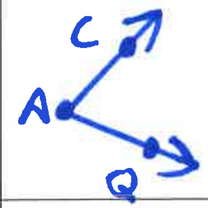
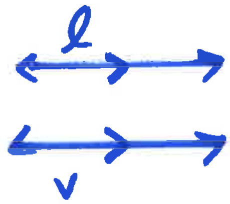
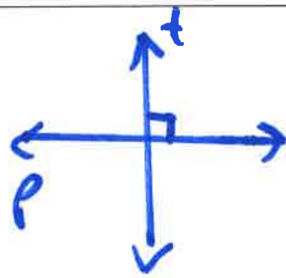
Measurement.

Design buildings, spatial analysis, proofs.

Points, lines, and planes are the building blocks of geometry.

Draw a representation for each of the following and fill in the appropriate notation on the chart below.

Description	Representation	Notation
A ***point*** is a precise location or place on a plane. It is usually represented by a dot.		point A
A ***line*** is a straight path that continues in both directions forever. Lines are one-dimensional.		line l $\overleftrightarrow{PQ}$
A ***plane*** is a flat, two-dimensional object. It has no thickness and extends forever.		Plane R
A ***line segment*** is a portion of a line located between two points.		$\overline{AB}$
A ***ray*** is piece of a line that starts at one point and extends infinitely in one direction.		$\overrightarrow{CD}$

Definition	Representation	Notation
An ***angle*** is formed by two rays with the same endpoint.		$\angle CAQ$
The point where the rays meet is called the ***vertex***.		Vertex M.
Parallel lines are two lines on the same plane that do not intersect.		line $l \parallel$ line v
Perpendicular lines are two intersecting lines that form a 90° angle.		line $t \perp$ line p

What can you say about multiple points on a line segment?

They all lie on the same line segment

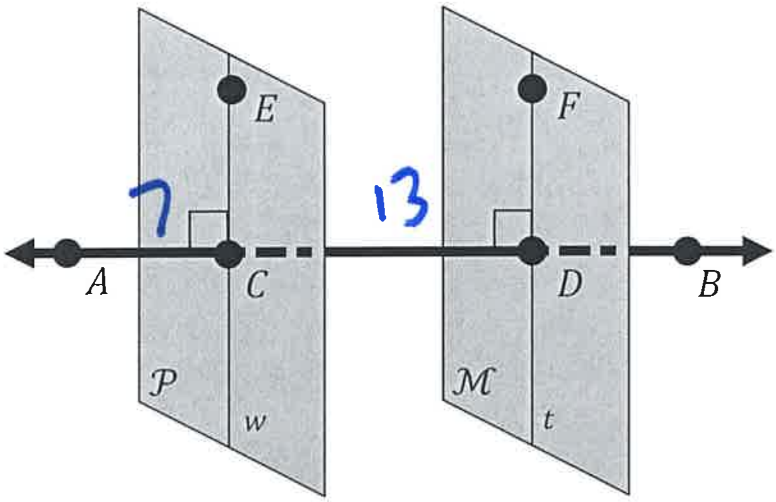
TAKE NOTE!
Postulates & Theorems

Segment Addition Postulate

If three points, $A, B,$ and C, are collinear and B is between A and C, then $AB + BC = AC$.

Let's Practice!

1. Consider the diagram below with parallel planes $\mathcal{P}$ and $\mathcal{M}$.



a. Give at most 3 names that represents the figure in the diagram above.

Figure	Name(s) denoted in diagram
Point	Point A, Point B, Point C
Line	line w, line t, $\overleftrightarrow{AC}$
Line Segment	$\overline{AC}$, $\overline{CD}$, $\overline{CE}$
Plane	Plane w, Plane t
Ray	$\overrightarrow{CA}$, $\overrightarrow{DB}$, $\overrightarrow{CB}$
Angle	$\angle ACE$, $\angle ECD$, $\angle FDB$
Parallel Lines	line w $\parallel$ line t
Perpendicular Lines	$\overleftrightarrow{CD} \perp \overleftrightarrow{EC}$, $\overleftrightarrow{CD} \perp \overleftrightarrow{FD}$
Segment Addition Postulate	$AC + CD = AD$

b. Point C lies between points A and D. If $AC = 7$ inches and $CD = 13$ inches, what is the ~~measure of $\overline{AD}$?~~

$AC + CD = AD$ AD = 20 inches

$7 + 13 = AD$

2. Point D lies between points P and Q. $PD = 3x + 6$. $DQ = 2x + 4$. $PQ = 30$. What is the measure of $\overline{PD}$?

$DQ = 2(4) + 4 = 12$

$PD + DQ = PQ$

$3x + 6 + 2x + 4 = 30$

$5x + 10 = 30$

$5x = 20$

$x = 4$

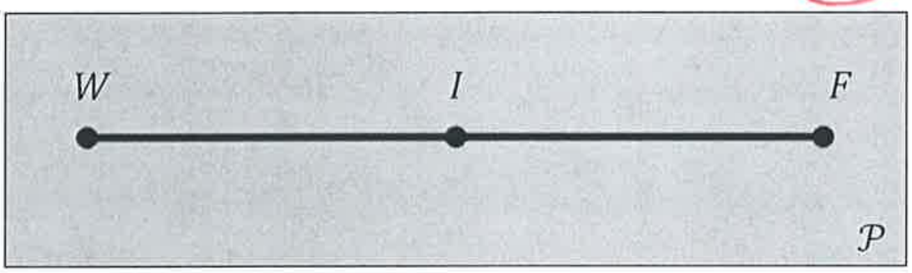
$PD = 3(4) + 6$

$= 12 + 6$

$= 18$

Try It!

3. Consider the diagram below.



T F

a. Determine if the following statements are true or false.

F Points W and F define a ray.
F $WI = WF$ by the Segment Addition Postulate.
T Points W, I, and F are collinear.
T Points W, I, and F are coplanar.

b. Point I lies between points W and F. $WI = 7x - 3$. $IF = 2x + 4$. $WF = 15x - 21$. What is the measure of $\overline{WF}$?

$WI + IF = WF$

$7x - 3 + 2x + 4 = 15x - 21$

$9x + 1 = 15x - 21$

$1 = 6x - 21$

$22 = 6x$

$\frac{22}{6} = x$

$x = \frac{11}{3}$

$WF = 15x - 21$

$WF = 15\left(\frac{11}{3}\right) - 21$

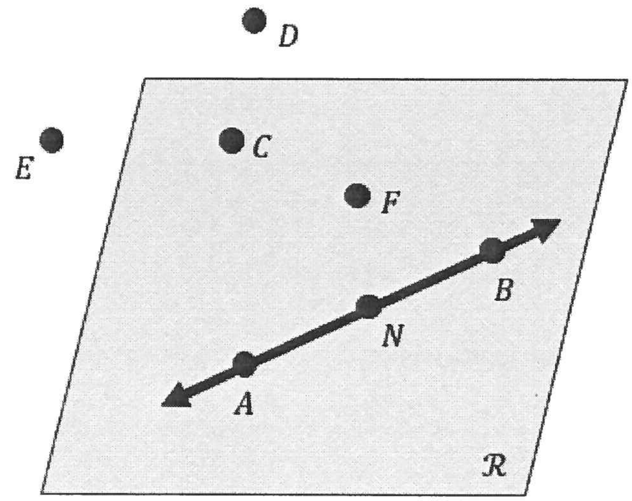
$= 55 - 21$

$WF = 34$

Section 1 – Topic 2
Basics of Geometry – Part 2

Let's Practice!

1. Consider the figure below.

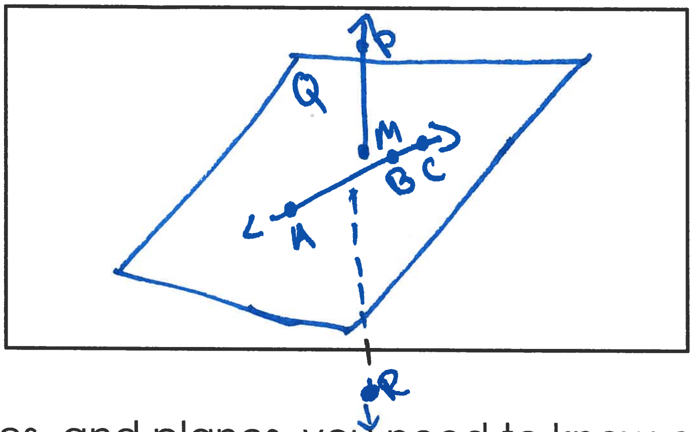


Select all the statements that apply to this figure.

☐ A, B, C, and D are coplanar in $\mathcal{R}$.
☐ A, B, C, and F are collinear.
☑ A, B, and N are collinear and coplanar in $\mathcal{R}$.
☑ B lies on $\overleftrightarrow{AN}$.
☑ A, C and F are coplanar in $\mathcal{R}$.
☐ C, D, E and F lie on $\mathcal{R}$.
☑ $AN + NB = AB$

Try It!

2. Plane Q contains $\overline{AB}$ and $\overrightarrow{BC}$, and it also intersects $\overleftrightarrow{PR}$ only at point M. Use the space below to sketch plane Q.



For points, lines, and planes, you need to know certain postulates.

STUDY EDGE TIP

A ***postulate*** is a statement that we take to be automatically true. We do not need to prove that a postulate is true because it is something we assume to be true.

Let's examine the following postulates A through F.

A. Through any two points there is exactly one line.

B. Through any three non-collinear points there is exactly one plane.

C. If two points lie in a plane, then the line containing those points will also lie in the plane.

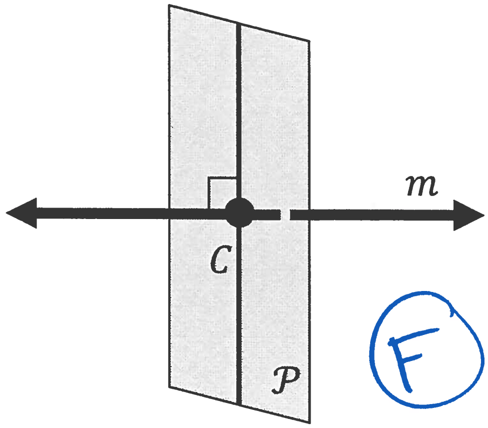
D. If two lines intersect, they intersect in exactly one point.

E. If two planes intersect, they intersect in exactly one line.

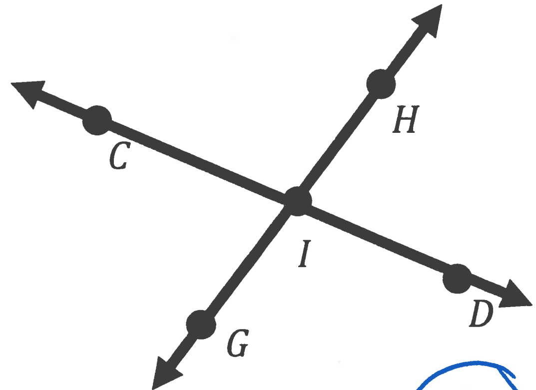
F. Given a point on a plane, there is one and only one line perpendicular to the plane through that point.

Let's Practice!

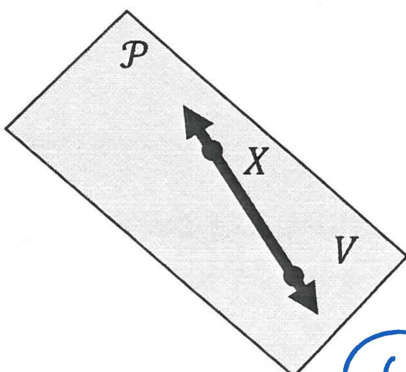
3. Use postulates A through F to match each visual representation with the correct postulate.



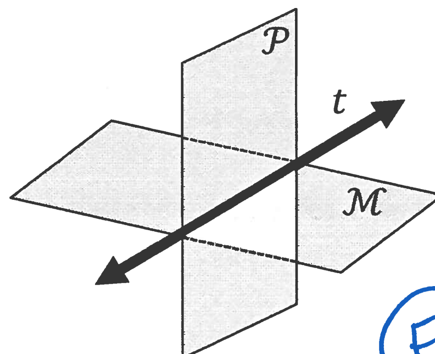
F



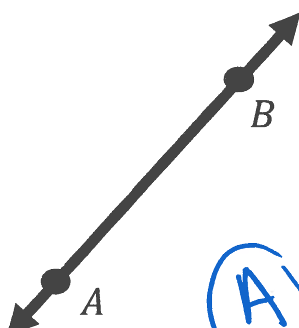
D



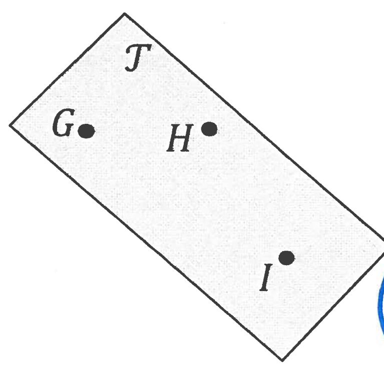
C



E



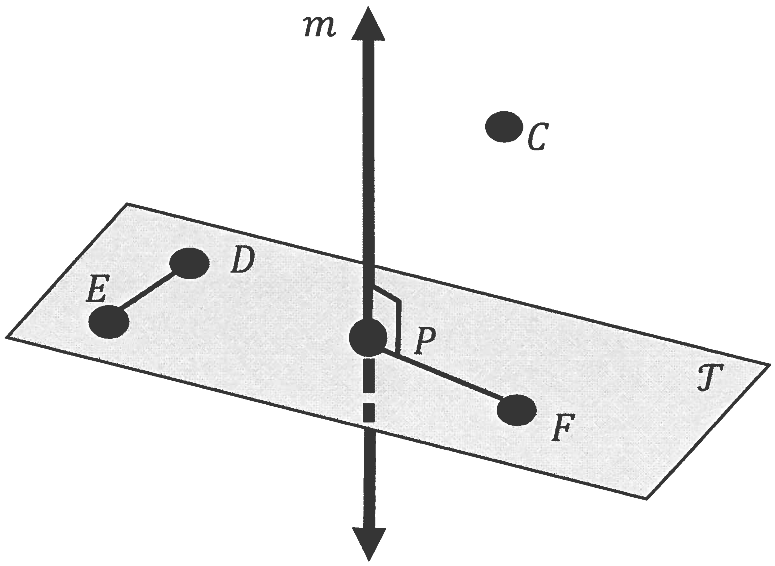
A



B

BEAT THE TEST!

1. Consider the following figure.



Select all the statements that apply to this figure.

- ☑ m is perpendicular through P to $\mathcal{T}$.
- ☐ C, D, E, and F are coplanar in $\mathcal{T}$.
- ☐ D, P, and F are collinear.
- ☐ $\overline{FC}$ is longer than $\overline{DF}$.
- ☑ $\overline{DE}$ and $\overline{PF}$ are coplanar in $\mathcal{T}$.

Section 1 – Topic 3
Introduction to Proofs

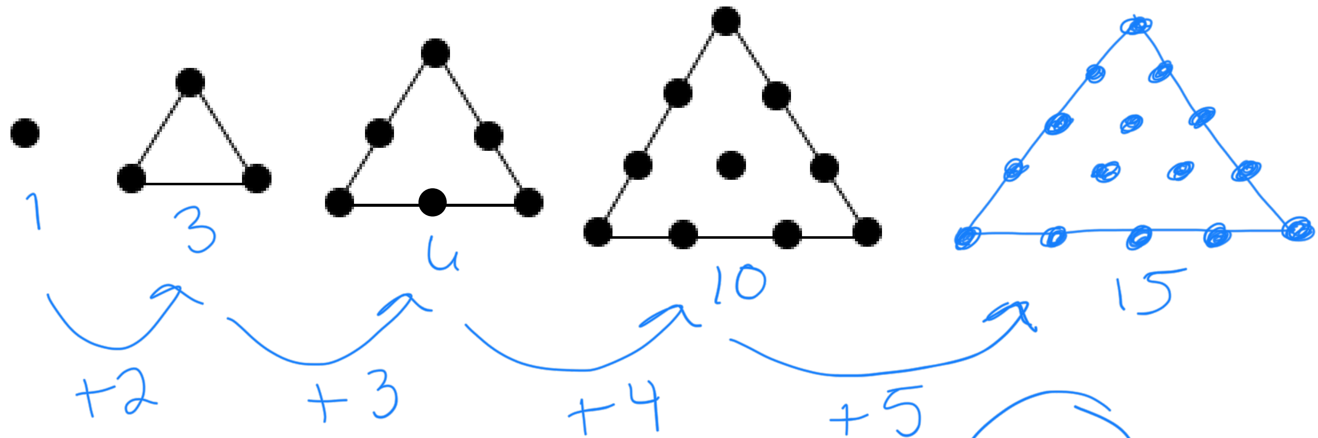
What are the next two terms in the following sequence?

$$5, 7, 11, 17, 25, \ldots \; 35 \quad 47$$

+2 +4 +6 +8 +10 +12

35 47

If the following pattern continues, how many dots will the fifth figure have?



15

Inductive reasoning is a type of reasoning that reaches conclusions based on a pattern.

A conjecture is a statement that is based on inductive reasoning but has not yet been shown to be true.

Make a conjecture: Based on the table, how many llamas would you expect the farm to have in year 7?

Year	Number of llamas at Sunny Day Farm
1	6
2	14
3	22
4	30

Conjecture: Each year, the number of llamas increase by 8. In year 7, I would expect 54 llamas.

STUDY EDGE TIP

To show that a conjecture is true, prove it is true for all cases, not just a few.

A counterexample is an example that shows a statement or conjecture is false.

What is a counterexample that shows the statement, "*If a number is a prime number, then the number is an odd number,*" is false?

Prime #s: 2, 3, 5, 7, 11, ...

The number 2 is prime and even.

Read the following statement. What can you logically conclude?

If $m\angle A$ is less than $90°$, then $\angle A$ is an acute angle.
$m\angle A = 85°$.

Since $m\angle A < 90°$, then $\angle A$ is an acute angle.

Deductive reasoning is a type of reasoning using given and previously known facts to reach a logical conclusion.

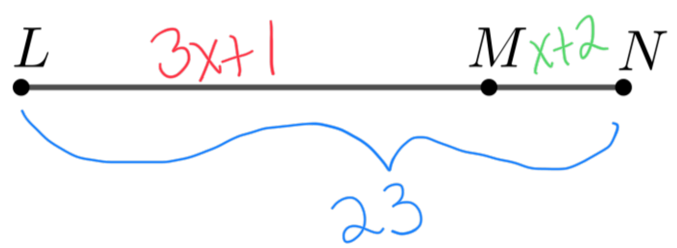
In this course, we will use deductive reasoning to prove statements. There are three different types of proofs:

Type of Proof	**Definition**
Two-Column	uses a table and explicitly places the statements in the first column and the reasoning in the second column
Paragraph	the statements and their reasoning are written together in a logical order in paragraph form
Flow Chart	a concept map where statements are placed in the boxes and the reason for each statement are placed under the box

Let's Practice!

1. Complete the two-column proof to prove that $x = 5$.

Given: $LM = 3x + 1$
$MN = x + 2$
$LN = 23$
Prove: $x = 5$



Statements	**Reasons**
1. $LM = 3x + 1$ $MN = x + 2$ $LN = 23$	1. Given
2. $3x+1 + x+2 = 23$	2. Segment Addition Postulate
3. $4x + 3 = 23$	3. Equivalent Equation
4. $4x+3-3 = 23-3$	4. Addition Property of Equality
5. $4x = 20$	5. Equivalent Equation
6. $(\frac{1}{4})4x = (\frac{1}{4})20$	6. Multiplication Property of Equality
7. $x = 5$	7. Equivalent Equation

What will the *first* row of a two-column proof always be?

The given statement(s).

What will the *last* row of a two-column proof always be?

The statement you are trying to prove.

2. The given figure is a square. The expression represents the area of the square. Use a paragraph proof to show that the length of one side of the square is $(2x + 3)$.

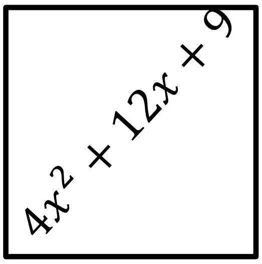


Figure is a square.

Given: Area $= 4x^2 + 12x + 9$

Prove: side of square $= 2x+3$

We are given the area of the square is represented by the expression $4x^2+12x+9$. By definition of a square, the area of a square is the length of a side squared. We can factor $4x^2+12x+9=(2x+3)^2$. Therefore, the side of the given square can be represented by the expression $2x+3$.

3. Use the word bank to prove the conditional using a flow chart proof.

$$\text{If } \frac{3x}{x+5} = 2, \text{ then } x = 10.$$

~~Given~~ Subtraction Property $\cancel{\frac{3x}{x+5} = 2}$

$\cancel{3x = 2(x + 5)}$ ~~Distributive Property~~

$\cancel{x = 10}$

$\cancel{3x = 2x + 10}$

~~Multiplication Property~~

START → $\boxed{\frac{3x}{x+5} = 2}$ → $\boxed{3x=2(x+5)}$ → $\boxed{3x=2x+10}$ → $\boxed{x=10}$ → END

$\frac{3x}{x+5} = 2$	$3x=2(x+5)$	$3x=2x+10$	$x=10$
Given	Multiplication Property of Equality	Distributive Property of Equality	Subtraction Property of Equality

Try It!

$\{1, 2, 3, 4, 5, \ldots\}$

4. When a natural number is added to three and the sum is divided by two, the quotient will be an even number.

Which of the following is a counterexample to the statement above?

Ⓐ $\frac{13+3}{2} = 8$, which is an even number. example

Ⓑ $\frac{12}{2} + 3 = 9$, which is not an even number. not correct rule

Ⓒ $\frac{3+4}{2} = \frac{7}{2}$, which is not an even number.

Ⓓ The statement is correct. There is no counterexample.

BEAT THE TEST!

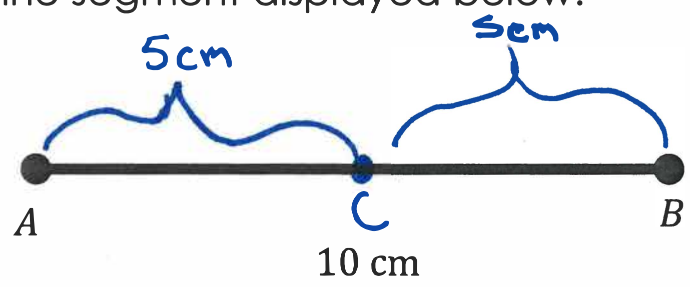
1. Consider the diagram below and finish the two-column proof to show $AC = BD$.

Given: $AB = CD$
Prove: $AC = BD$

Statements	Reasons
1. $AB = CD$	1. Given
2. $BC = BC$	2. Reflexive Property
3. $AB + BC = BC + CD$	3. Addition Property of Equality
4. $AB + BC = AC$ $BC + CD = BD$	4. Segment Addition Postulate
5. $AC = BD$	5. Substitution

Section 1 – Topic 4
Midpoint and Distance in the Coordinate Plane – Part 1

Consider the line segment displayed below.



The length of $\overline{AB}$ is 10 centimeters.

- Distance is an amount of space (in certain units) between two points on a plane.

Draw a point halfway between point A and point B. Label this point C.

What is the length of $\overline{AC}$? 5cm

What is the length of $\overline{CB}$? 5 cm

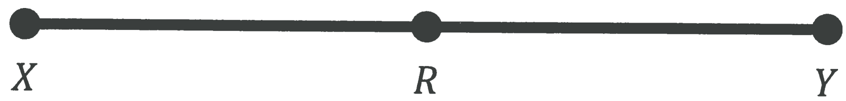
Point C is called the midpoint of $\overline{AB}$.

Why do you think it's called the midpoint?

It's in the middle of A and B (halfway from each point)

Let's Practice!

1. Consider $\overline{XY}$ with midpoint R.



a. What can be said of $\overline{XR}$ and $\overline{RY}$?

$\overline{XR} \cong \overline{RY}$

distance $\overline{XR}$ = distance $\overline{RY}$

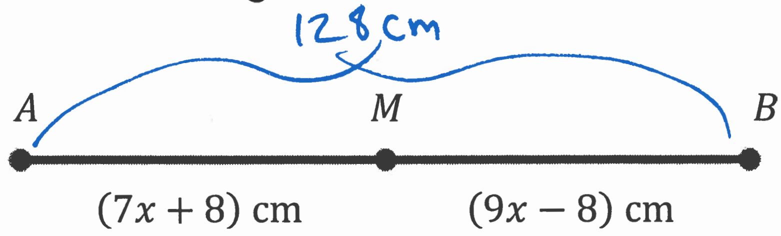
b. If $\overline{XR}$ is $(2x + 5)$ inches long and $\overline{RY}$ is 22 inches long, what is the value of x?

$$2x + 5 = 22$$
$$-5 \quad -5$$

$$\frac{2x}{2} = \frac{17}{2}$$

$$\boxed{x = 8.5}$$

2. Consider the line segment below.



a. If $\overline{AB}$ is 128 centimeters long, what is x?

$(7x+8) + (9x-8) = 128$

$7x + 8 + 9x - 8 = 128$

$\frac{16}{16} = \frac{128}{16}$ $x = 8$

$\boxed{x \text{ is } 8\text{cm}}$

b. What is the length of $\overline{AM}$?

$7x + 8 \to 7(8) + 8$

$= 56 + 8$

$= 64$

$\boxed{\overline{AM} = 64\text{cm}}$

c. What is the length of $\overline{BM}$?

$9x - 8 \to 9(8) - 8$

$= 72 - 8$

$= 64$

$\boxed{\overline{BM} = 64\text{cm}}$

d. Is point M the midpoint of $\overline{AB}$? Justify your answer.

Yes. $\overline{AM} = \overline{MB}$

$x = 8$ $x = 8$

$7(8) + 8 = 9(8) - 8$

$64 = 64$

Try It!

3. Diego and Anya live 72 miles apart. They both meet at their favorite restaurant, which is $(16x - 3)$ miles from Diego's house and $(5x + 2)$ miles from Anya's house.

   Diego argues that in a straight line distance, the restaurant is halfway between his house and Anya's house. Is Diego right? Justify your reasoning. Diego is not right

Diego Rest. Anya's

$16x - 3$ $5x + 2$

Find x. → $\boxed{x = 73/21}$

$(5x+2) + (16x-3) = 72$

$5x + 16x + 2 - 3 = 72$

$21x - 1 = 72$

$+1 \quad +1$

$\frac{21x}{21} = \frac{73}{21}$

Verify midpoint:

$16x - 3 = 5x + 2$

$-5x + 3 \quad -5x + 3$

$\frac{11x}{11} = \frac{5}{11}$

$x = 5/11$

$\boxed{5/11 \neq 73/21}$

If midpoint, then:

$5x + 2 = 36$

$5(73/21) + 2 = 36$

$17.38095 + 2 = 36$

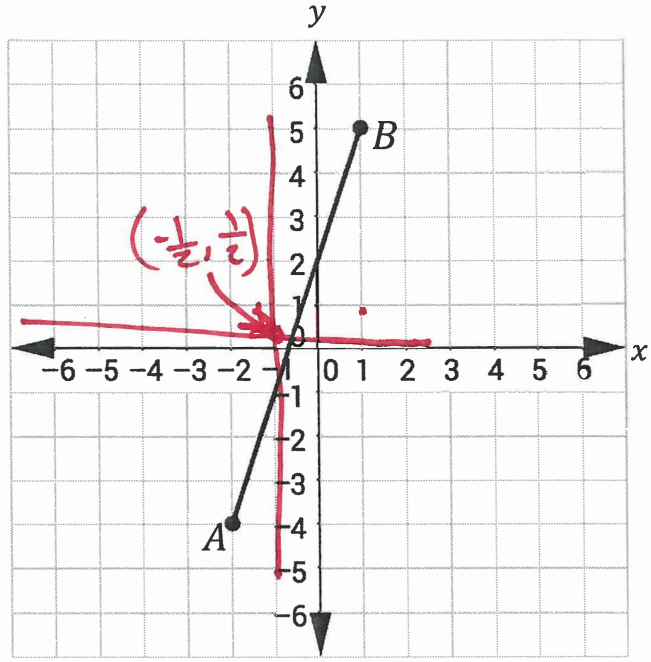
$\boxed{19.38095 \neq 36}$

Midpoint and ***distance*** can also be calculated on a coordinate plane.

The coordinate plane is a plane that is divided into __4__ regions (called quadrants) by a horizontal line (__x-axis__) and a vertical line (__y-axis__).

- ➢ The location, or coordinates, of a point are given by an ordered pair, __(x, y)__.

Consider the following graph.



Name the ordered pair that represents point A.

(-2, -4)

Name the ordered pair that represents point B.

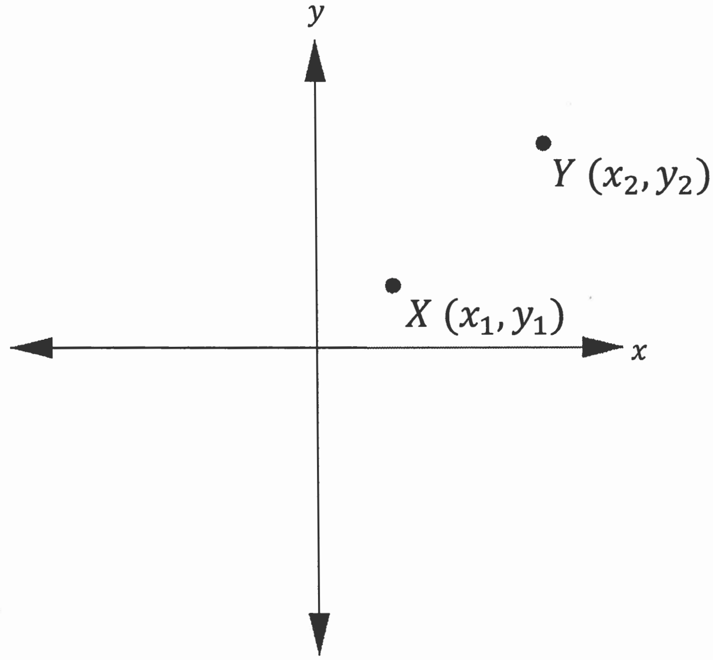
(1, 5)

How can we find the midpoint of this line?

Find the midpoint of x-coordinates and the midpoint of y-coordinates

The midpoint of $\overline{AB}$ is ($-\frac{1}{2}$, $\frac{1}{2}$).

Let's consider points X and Y on the coordinate plane below.



Write a formula that can be used to find the midpoint of any two given points.

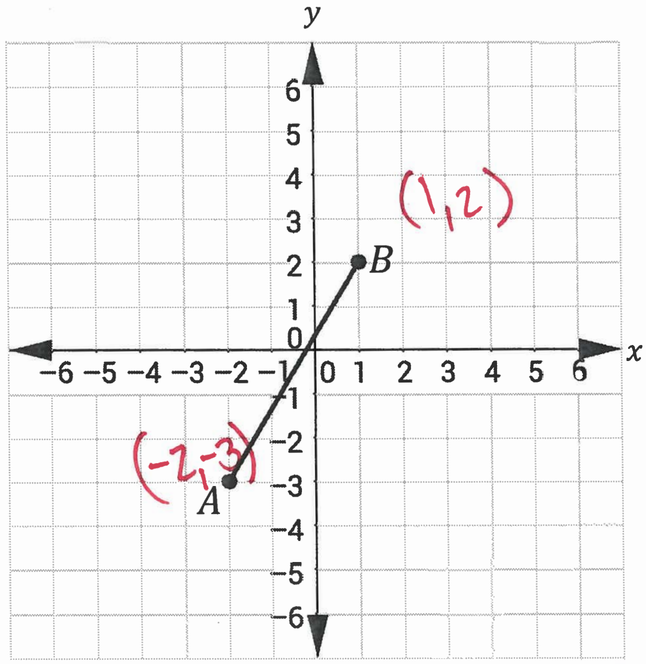
Midpoint for x coordinates: $\frac{x_1 + x_2}{2}$

Midpoint for y-coordinates: $\frac{y_1 + y_2}{2}$

Midpoint for $\overline{XY}$ $= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$

Let's Practice!

4. Consider the line segment in the graph below.



Find the midpoint of $\overline{AB}$.

$x_1 = -2$

$x_2 = 1$

$y_1 = -3$

$y_2 = 2$

$$\text{Midpoint}_{\overline{AB}} = \left(\overset{M_x}{\frac{-2+1}{2}}, \overset{M_y}{\frac{-3+2}{2}} \right) = \boxed{\left(\overset{M_x}{-\frac{1}{2}}, \overset{M_y}{-\frac{1}{2}} \right)}$$

5. M is the midpoint of $\overline{CD}$. C has coordinates $(-1,-1)$ and M has coordinates $(3,5)$. Find the coordinates of D.

$x_1 = -1$ $\quad M_x = \frac{x_1 + x_2}{2}$

$x_2 = 7$ $\quad (2)\, 3 = \frac{-1 + x_2}{2}\, (2)$

$M_x = 3$

$6 = -1 + x_2$

$+1 \quad +1$

$7 = x_2$

$y_1 = -1$ $\quad M_y = \frac{y_1 + y_2}{2}$

$y_2 = 11$ $\quad (2)\, 5 = \frac{-1 + y_2}{2}\, (2)$

$M_y = 5$

$10 = -1 + y_2$

$+1 \quad +1$

$11 = y_2$

$M_x = 7$

$\boxed{D(7, 11)}$

Try It!

6. P has coordinates $(2, 4)$. Q has coordinates $(-10, 12)$. Find the midpoint of $\overline{PQ}$.

$x_1 = 2$

$x_2 = -10$ $\quad M_x =$

$y_1 = 4$

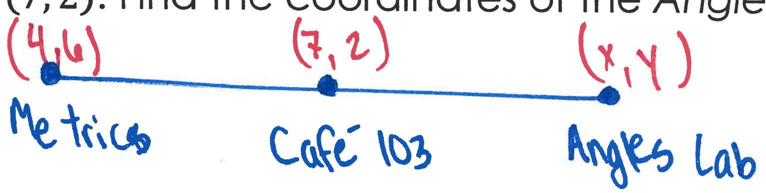
$y_2 = 12$ $\quad M_y =$

$$\text{Midpoint}_{\overline{PQ}} = \left(\overset{M_x}{\frac{2+(-10)}{2}}, \overset{M_y}{\frac{4+12}{2}}\right)$$

$$= \left(\frac{-8}{2}, \frac{16}{2}\right)$$

$$\boxed{= (-4, 8)}$$

7. *Café 103* is collinear with and equidistant from the *Metrics School* and the *Angles Lab*. The *Metrics School* is located at point $(4, 6)$ on a coordinate plane, and *Café 103* is at point $(7, 2)$. Find the coordinates of the *Angles Lab*.



$M_x = \frac{x_1 + x_2}{2}$ $\qquad M_y = \frac{y_1 + y_2}{2}$

$(2)\,7 = \frac{4 + x_2}{2}\,(2)$ $\qquad (2)\,2 = \frac{6 + y_2}{2}\,(2)$

$14 = 4 + x_2$ $\qquad 4 = 6 + y_2$

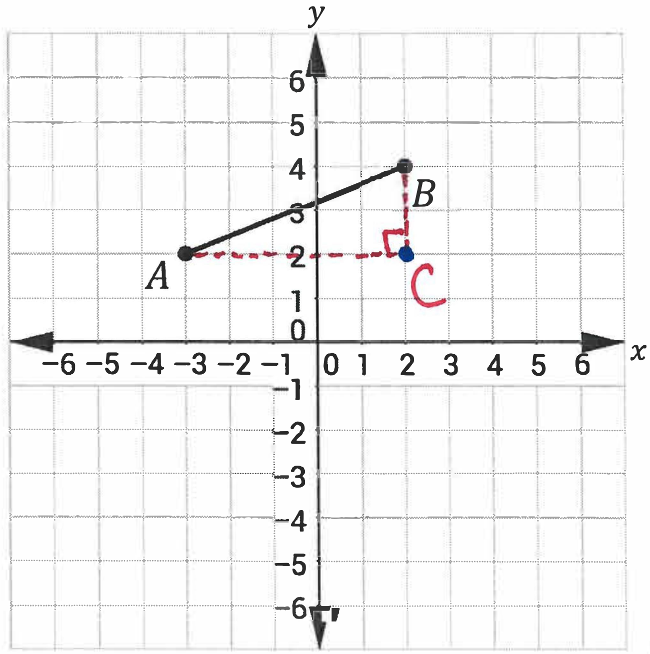
$-4 \quad -4$ $\qquad -6 \quad -6$

$10 = x_2$ $\qquad -2 = y_2$

$\boxed{\text{Angles Lab is located at } (10, -2).}$

Section 1 – Topic 5
Midpoint and Distance in the Coordinate Plane – Part 2

Consider $\overline{AB}$ below.



Draw point C on the above graph at $(2, 2)$.

What is the length of $\overline{AC}$?

5 units

What is the length of $\overline{BC}$?

2 units

Triangle ABC is a right triangle. Use the Pythagorean Theorem to find the length of $\overline{AB}$.

$5^2 + 2^2 = (\overline{AB})^2$

$25 + 4 = (\overline{AB})^2$

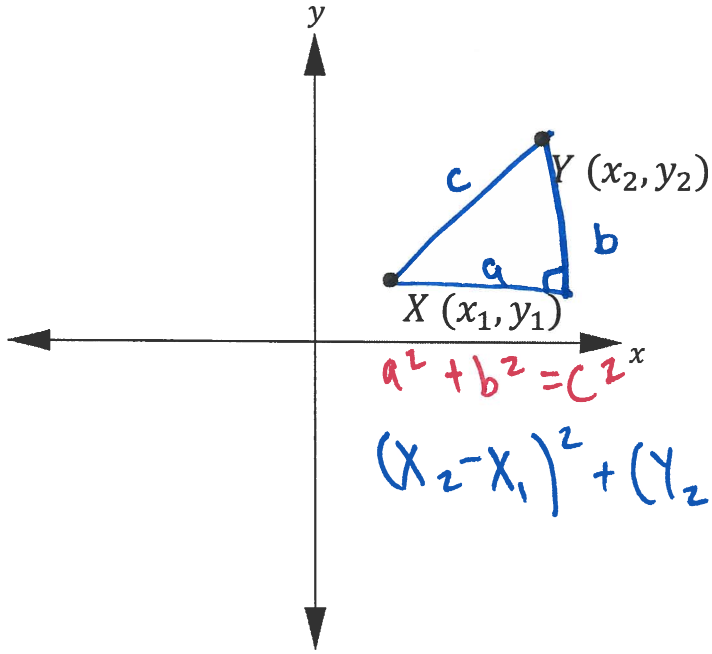
$29 = (\overline{AB})^2$

$\sqrt{29} = \overline{AB}$

$\overline{AB} = \sqrt{29}$

$\overline{AB} = 5.4$ units

Let's consider the figure below.



$a^2+b^2=c^2$

$(x_2-x_1)^2+(y_2-y_1)^2=d^2$

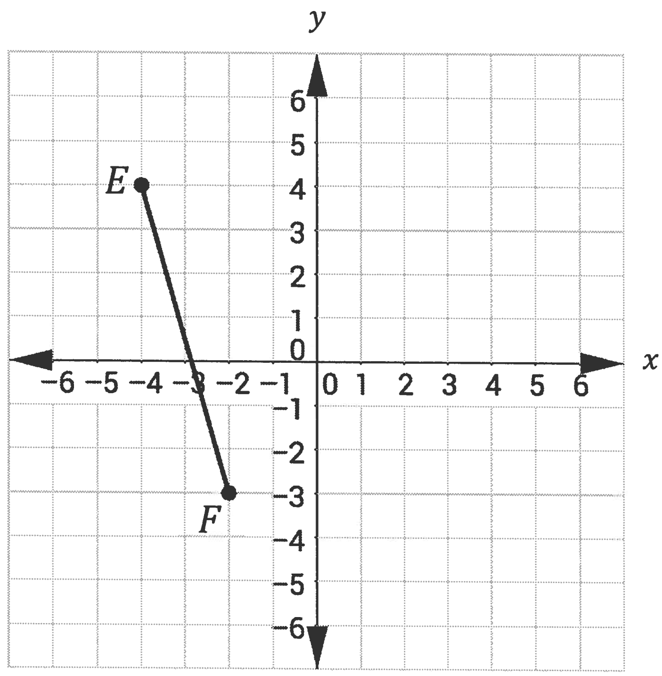
Write a formula to determine the distance of any line segment.

$\sqrt{d^2}=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

Let's Practice!

1. Find the length of $\overline{EF}$.



$x_1 = -4$ $\quad$ $y_1 = 4$

$x_2 = -2$ $\quad$ $y_2 = -3$

$$d_{\overline{EF}} = \sqrt{(-4-(-2))^2 + (4-(-3))^2}$$

$$d_{\overline{EF}} = \sqrt{(-4-2)^2 + (4+3)^2}$$

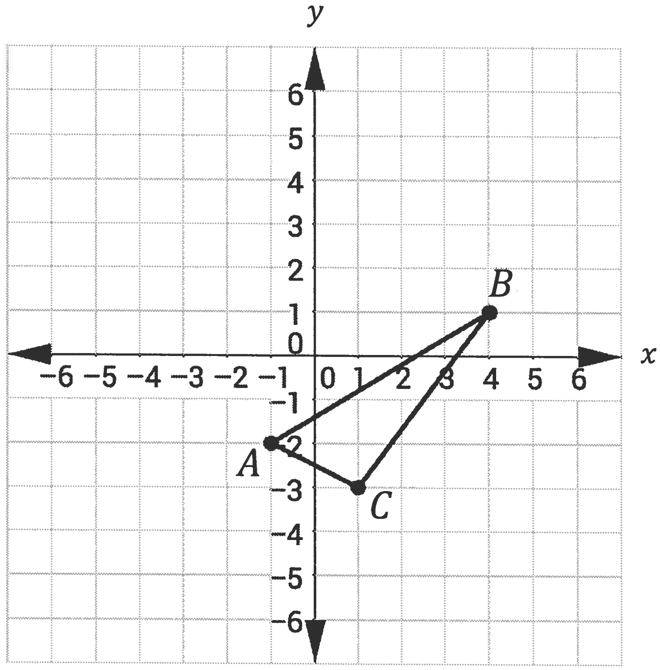
$$d_{\overline{EF}} = \sqrt{(-2)^2 + (7)^2}$$

$$d_{\overline{EF}} = \sqrt{4 + 49}$$

$$d_{\overline{EF}} = \sqrt{53} \approx \boxed{7.3\text{units}}$$

Try It!

2. Consider triangle ABC graphed on the coordinate plane.



Find the perimeter of triangle ABC. $\longrightarrow \overline{AB} + \overline{BC} + \overline{AC}$

$$d_{\overline{AB}} = \sqrt{(-1-4)^2 + (-2-1)^2}$$
$$= \sqrt{(-5)^2 + (-3)^2}$$
$$= \sqrt{25 + 9}$$
$$= \sqrt{34}$$
$$d_{\overline{AB}} \approx 5.8 \text{ units}$$

$$d_{\overline{BC}} = \sqrt{(1-4)^2 + (-3-1)^2}$$
$$= \sqrt{(-3)^2 + (-4)^2}$$
$$= \sqrt{9 + 16}$$
$$= \sqrt{25}$$
$$d_{\overline{BC}} = 5 \text{ units}$$

$$d_{\overline{AC}} = \sqrt{(-1-1)^2 + (-2-(-3))^2}$$
$$= \sqrt{(-2)^2 + (1)^2}$$
$$= \sqrt{4+1}$$
$$= \sqrt{5}$$
$$d_{\overline{AC}} \approx 2.2 \text{ units}$$

$\boxed{\text{Perimeter}_{\triangle ABC} \rightarrow 5.8 + 5 + 2.2 = 13 \text{ units}}$

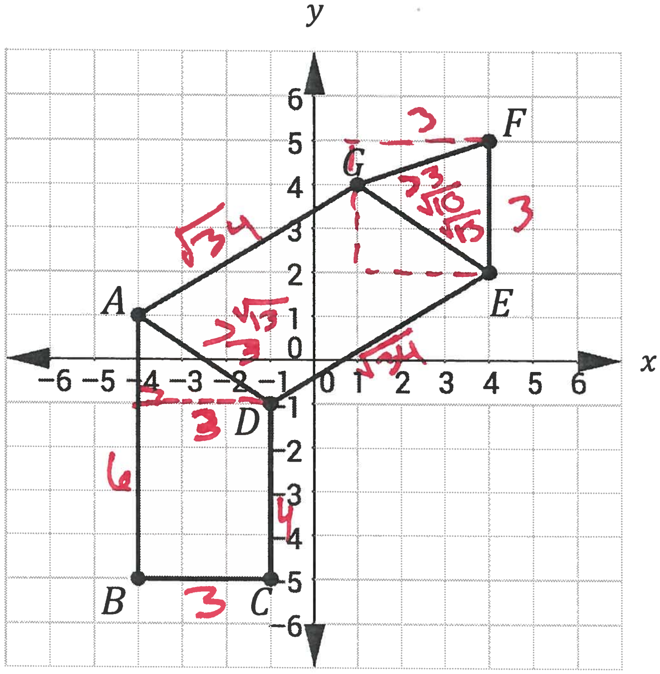
BEAT THE TEST!

1. Consider the following figure.

$P_{ABCD} = 6+4+3 + \sqrt{13}$
$= 13 + \sqrt{13}$
≈ 16.61

$P_{ADEG} = 2\sqrt{34} + 2\sqrt{13}$
$\approx 11.66 + 7.21$
≈ 18.87

$P_{\Delta EFG} = \sqrt{13} + \sqrt{10} + 3$
$= 9.7678$



$M_{\overline{AC}} = \left(\frac{-4+1}{2}, \frac{1+4}{2}\right)$

$M_{\overline{AC}} = \left(\frac{-3}{2}, \frac{5}{2}\right)$

Which of the following statements are true? Select all that apply.

- ☑ The midpoint of $\overline{AG}$ has coordinates $\left(-\frac{3}{2}, \frac{5}{2}\right)$.
- ☐ $\overline{DE}$ is exactly 5 units long.
- ☐ $\overline{AD}$ is exactly 3 units long.
- ☑ $\overline{FG}$ is longer than $\overline{EF}$.
- ☑ The perimeter of quadrilateral $ABCD$ is about 16.6 units.
- ☑ The perimeter of quadrilateral $ADEG$ is about 18.8 units.
- ☐ The perimeter of triangle EFG is 9 units.

Section 1 – Topic 6
Partitioning a Line Segment – Part 1

What do you think it means to ***partition***?

A division into or distribution in portions or shares.

How can a line segment be partitioned?

Break in pieces w/ points inside the line segment

In the previous section, we worked with the midpoint, which partitions a segment into a $1:1$ ratio.

> **STUDY EDGE TIP**
>
> A ***ratio*** compares two numbers. A $1:1$ ratio is stated as, or can also be written as, "1 to 1".

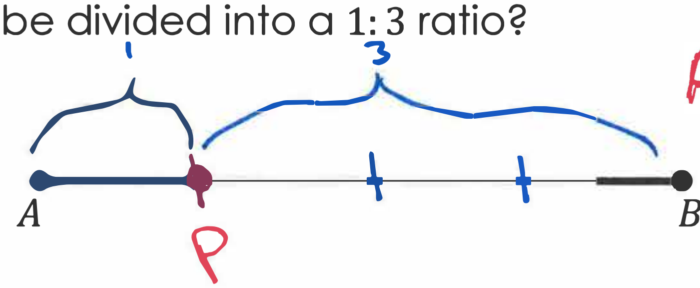
2 peices

Why does the midpoint partition a segment into a $1:1$ ratio?

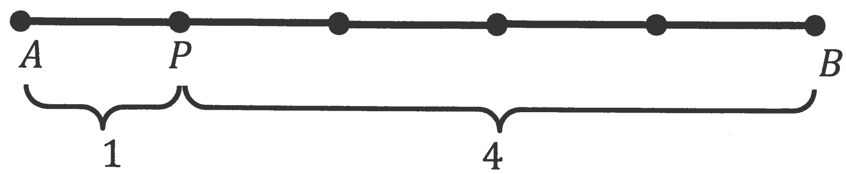
midpoint break segment into 2 equal pieces

How can $\overline{AB}$ be divided into a $1:3$ ratio?

Answer may very



Consider the following line segment where point P partitions the segment into a $1:4$ ratio.



How many sections are between points A and P?

1

How many sections are between points P and B?

4

How many sections are between points A and B?

5

In relation to $\overline{AB}$, how long is $\overline{AP}$?

1/5

In relation to $\overline{AB}$, how long is $\overline{PB}$?

4/5

Let's call these ratios, k, a fraction that compares a part to a whole.

If partitioning a directed line segment into two segments, when would your ratio k be the same for each segment? When would it differ? Same if partitioned with midpoint. Otherwise; it would be different.

The following formula can be used to find the coordinates of a given point that partitions a line segment into ratio k.

$$(x, y) = \left(x_1 + k(x_2 - x_1), y_1 + k(y_2 - y_1)\right)$$

Let's Practice!

1. What is the value of k used to find the coordinates of a point that partitions a segment into a ratio of $4:3$?

$4/7$

2. Determine the value of k if partitioning a segment into a ratio of $1:5$.

$1/6$

Try It!

3. Point A has coordinates $(2, 4)$. Point B has coordinates $(10, 12)$. Find the coordinates of point P that partitions $\overline{AB}$ in the ratio $3:2$. $k = 3/5$

Find Y_p:

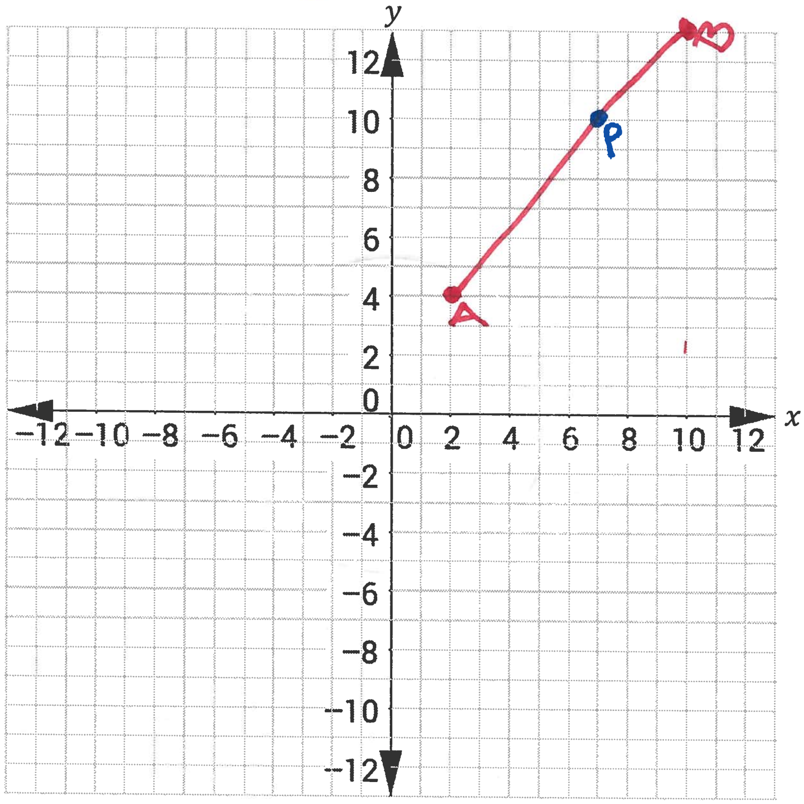
$2 + 3/5\,(10-2)$

$2 + 3/5\,(8)$

$2 + 24/5$

$34/5 = 6.8$

$X_p = 6.8$



Find Y_p:

$4 + 3/5\,(12-4)$

$4 + 3/5\,(8)$

$4 + 24/5$

$44/5 = 8.8$

$Y_p = 8.8$

$\boxed{P(6.8, 8.8}$

4. Points C, D, and E are collinear on $\overline{CE}$, and $CD : DE = \frac{3}{5}$. C is located at $(1, 8)$, D is located at $(4, 5)$, and E is located at (x, y). What are the values of x and y?

part part

$X_R \Rightarrow 1 + \frac{3}{8}(x-1) = 4$

$1 + \frac{3}{8}x \left(-\frac{3}{8}\right) = 4$

$\frac{5}{8} + \frac{3}{8}x = 4 - \frac{5}{8}$

$-\frac{5}{8} \quad \frac{3}{8}x = \frac{27}{8}$

$\boxed{x = 9}$

$Y_R \Rightarrow 8 + \frac{3}{8}(y-8) = 5$

$8 + \frac{3}{8}y \; -3$

$5 + \frac{3}{8}y = 5$

$-5 \qquad -5$

$\frac{3}{8}y = 0$

$\boxed{y = 0}$

Section 1 – Topic 7
Partitioning a Line Segment – Part 2

Consider $M, N,$ and P, collinear points on $\overline{MP}$.

What is the difference between the ratio $MN:NP$ and the ratio of $MN:MP$? Ratio MN : NP compares parts to parts. It compares the partitions. Ratio MN : MP compares parts to the whole; so the ratio equals the factor K.

What should you do if one of the parts of a ratio is actually the whole line instead of a ratio of two smaller parts or segments?

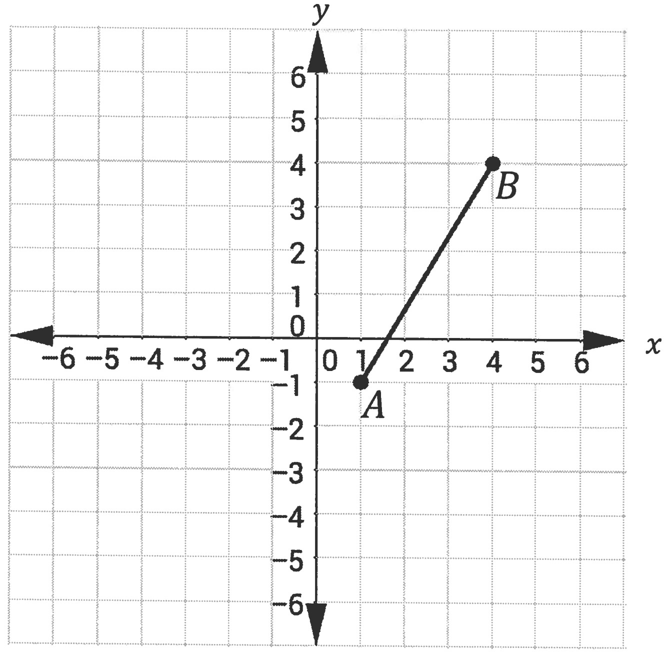
K = ratio of part-to-whole
but same algebraic method.

Let's Practice!

1. Points $P, Q,$ and R are collinear on $\overline{PR}$, and $PQ:PR = \frac{2}{3}$ (part:whole $= K$). P is located at the origin (0,0), Q is located at (x, y), and R is located at $(-12, 0)$. What are the values of x and y?

$$(x, y) = (x_1 + k(x_2 - x_1),\ y_1 + k(y_2 - y_1))$$
$$= (0 + \tfrac{2}{3}(-12 - 0),\ 0 + \tfrac{2}{3}(0 - 0))$$
$$= 0 + (-8),\ 0 + 0)$$
$$= (-8, 0) \longrightarrow \boxed{\begin{array}{l} x = -8 \\ y = 0 \end{array}}$$

2. Consider the line segment in the graph below.



a. Find the coordinates of point P that partition $\overline{AB}$ in the ratio $1:4$.

part : part $k = \frac{1}{5}$

$(x,y) = \left(1+\frac{1}{5}(4-1), -1+\frac{1}{5}(4-(-1))\right)$

$= \left(1+\frac{3}{5}, -1+1\right)$

$= \left(\frac{8}{5}, 0\right) \rightarrow \boxed{P\left(\frac{8}{5}, 0\right) \text{ or } P(1.6, 0)}$

b. Suppose A, R, and B are collinear on $\overline{AB}$, and $AR: AB = \frac{1}{4}$. What are the coordinates of R?

Part : whole $k = \frac{1}{4}$

$(x,y) = \left(1+\frac{1}{4}(4-1), -1+\frac{1}{4}(4-(-1))\right)$

$= \left(1+\frac{3}{4}, -1+\frac{5}{4}\right)$

$= \left(\frac{7}{4}, \frac{1}{4}\right) \rightarrow \boxed{R\left(\frac{7}{4}, \frac{1}{4}\right) \text{ or } R(1.75, 0.25)}$

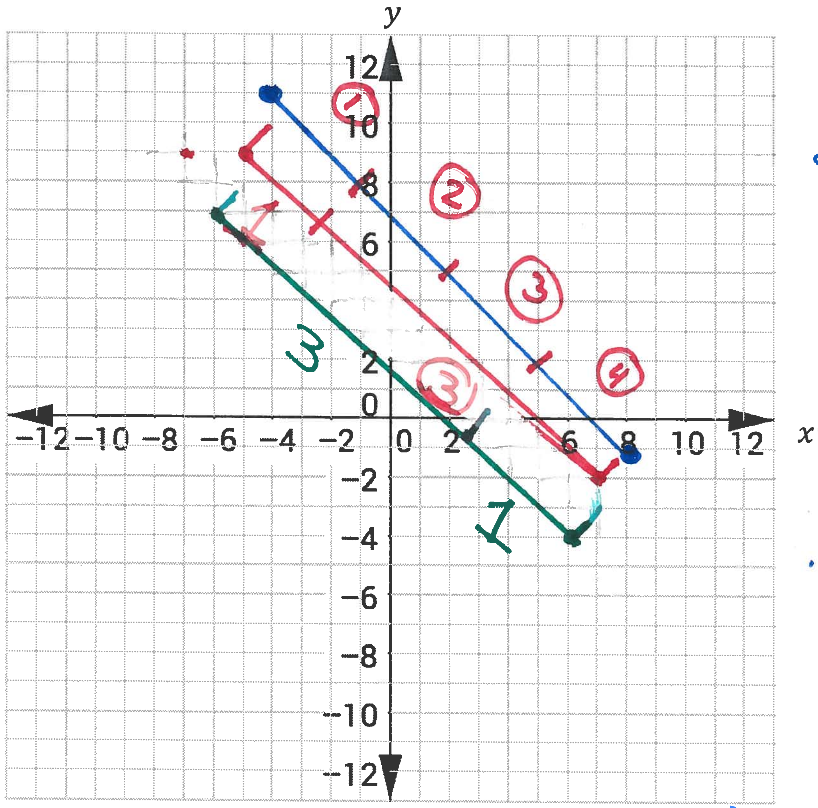
Try It!

3. $\overline{JK}$ in the coordinate plane has endpoints with coordinates $(-4, 11)$ and $(8, -1)$.

   a. Graph $\overline{JK}$ and find two possible locations for point M, so M divides $\overline{JK}$ into two parts with lengths in a ratio of $1:3$. *somtimes you can use the graph to partition.

   $k = \frac{1}{4}$

Possible Location #1 $M_1(-1,8)$



Possible Location #2 $M_2(5,2)$

*Verify using the algebraic method (suggest this!!!)

   b. Suppose J, P, and K are collinear on $\overline{JK}$, and $JP:JK = \frac{1}{3}$. What are the coordinates of P?

   part : whole

   $k = \frac{1}{3}$

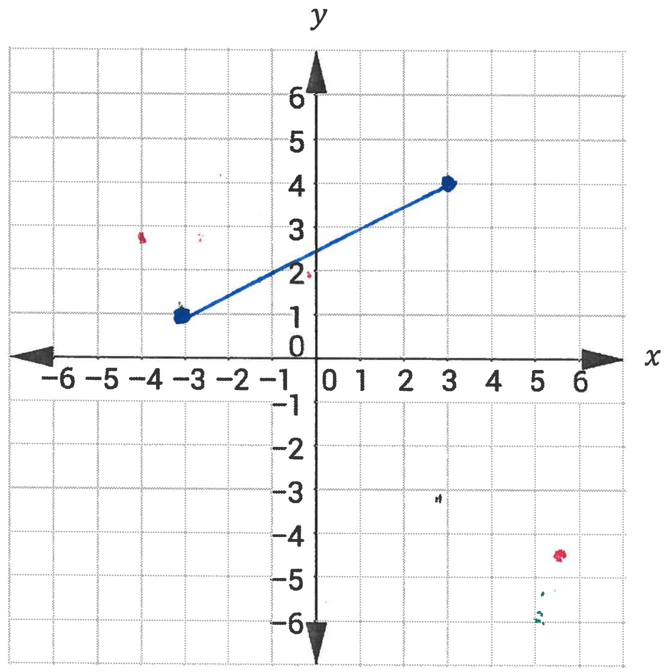
$$(x,y) = \left(-4 + \tfrac{1}{3}(-(-4)),\ 11 + \tfrac{1}{3}(-1-11)\right)$$
$$= (-4+4,\ 11+(-4))$$
$$= (0,7) \longrightarrow \boxed{P(0,7)}$$

*P(4,3) also works if you invert the x_2 and x_1.

BEAT THE TEST!

1. Consider the directed line segment from $A(-3, 1)$ to $Z(3, 4)$. Points $L, M,$ and N are on $\overline{AZ}$.

$L(-1, 2)$	$M\left(0, \frac{5}{2}\right)$	$N(1, 3)$



Complete the statements below.

The point <u>M</u> partitions $\overline{AZ}$ in a $1:1$ ratio.

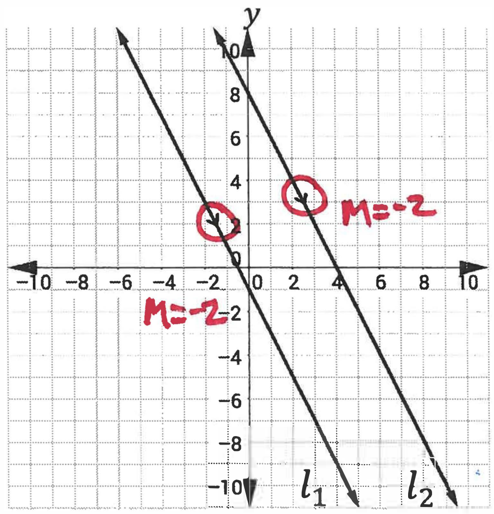
The point <u>L</u> partitions $\overline{AZ}$ in a $1:2$ ratio.

The point <u>N</u> partitions $\overline{AZ}$ in a $2:1$ ratio.

The ratio $AL: AZ =$ <u>$\frac{1}{3}$</u>.

Section 1 – Topic 8
Parallel and Perpendicular Lines – Part 1

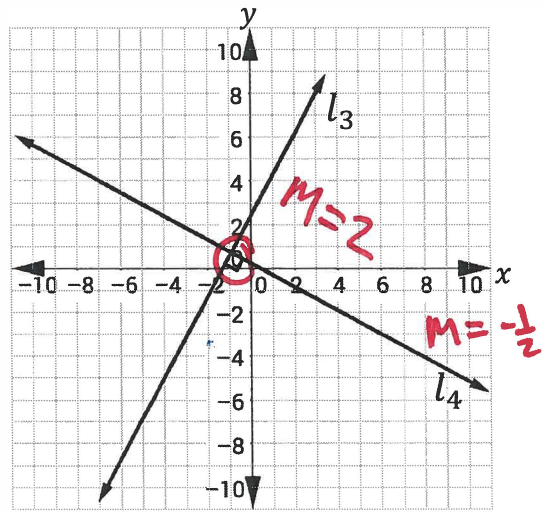
Graph A



These lines are parallel.

The symbol used to indicate ***parallel*** lines is $\parallel$.

Graph B



These lines are perpendicular.

The symbol used to indicate ***perpendicular*** lines is $\perp$.

Choose two points on each graph and use the slope formula, $\frac{y_2-y_1}{x_2-x_1}$, to verify your answers.

What do you notice about the slopes of the parallel lines?

Same

What do you notice about the slopes of the perpendicular lines? The product of their slopes is -1.

Negative Recipocal

What happens if the lines are are given in equation form instead of on a graph?

compare slopes of each equation

Let's Practice!

1. Indicate whether the lines are parallel, perpendicular, or neither. Justify your answer.

   a. $y = 2x$ and $6x = 3y + 5$

   $M=2$ $M=2$

   Parrallel, because slopes are the same

   b. $2x - 5y = 10$ and $10x + 4y = 20$

   $M = \frac{2}{5}$ $M = \frac{-10}{4} = -\frac{5}{2}$

   Perpendicular, because the product of the slopes is -1.

   c. $4x + 3y = 63$ and $12x - 9y = 27$

   $M = \frac{-4}{3}$. $M = \frac{4}{3}$ Neither

   d. $x = 4$ and $y = -2$

   $M=$ undefined $M=0$

   Perpendicular

Try It!

2. Write the letter of the appropriate equation in the column beside each item.

A. $x = -5$	**B.** $y = -\frac{1}{4}x + 1$	**C.** $3x - 5y = -30$	**D.** $x - 2y = -2$
undefined.		$M = \frac{3}{5}$	$M = \frac{1}{2}$

C	A line parallel to $y = \frac{3}{5}x + 2$	$M = \frac{3}{5}$
A	A line perpendicular to $y = 4$	$M = 0$
D	A line perpendicular to $4x + 2y = 12$	$M = -2$
B	A line parallel to $2x + 8y = 7$	$M = -\frac{1}{4}$

Let's Practice!

1. Write the equation of the line passing through $(-1, 4)$ and perpendicular to $x + 2y = 11$.

$M = -\frac{1}{2}$ $\quad M_{\perp} = 2$

$4 = 2(-1) + b$

$4 = -2 + b$

$6 = b \longrightarrow \boxed{y = 2x + 6}$

Try It!

2. Suppose the equation for line A is given by $y = -\frac{3}{4}x - 2$. If line A and line B are perpendicular and the point $(-4, 1)$ lies on line B, then write an equation for line B.

$M_{\perp} = \frac{4}{3}$

$1 = \frac{4}{3}(-4) + b$

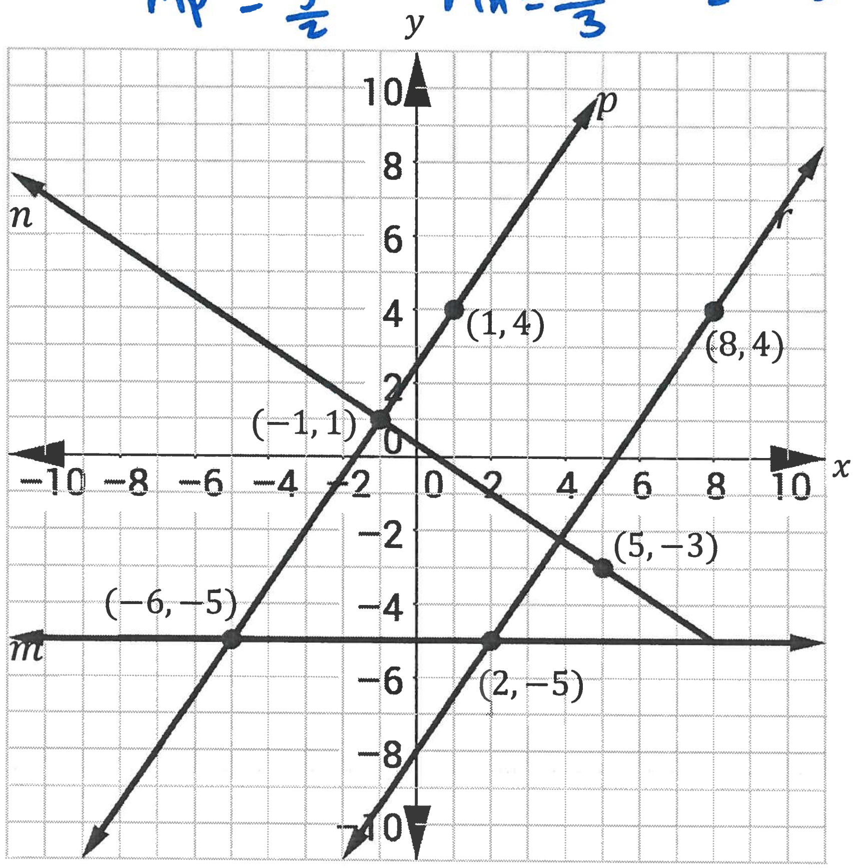
$1 = \frac{-16}{3} + b$

$\frac{19}{3} = b \longrightarrow \boxed{y = \frac{4}{3}x + \frac{19}{3}}$

3. Consider the graph below.

a. Name a set of lines that are ~~parallel~~ perpendicular. Justify your answer. line p $\perp$ line n $\frac{3}{2}\left(-\frac{2}{3}\right) = -1$

$m_p = \frac{3}{2}$, $m_n = \frac{-2}{3}$



b. Name a set of lines that are ~~perpendicular~~ parallel. Justify your answer.

line p $\parallel$ line r $m_p = \frac{3}{2}$ $m_r = \frac{3}{2}$

$m_p = m_r$

BEAT THE TEST!

1. The equation for line A is given by $y = -\frac{3}{4}x - 2$. Suppose line A is parallel to line B, and line T is perpendicular to line A. Point $(0, 5)$ lies on both line B and line T.

A B

point : (0, 5) $y = mx + b$

$M = \frac{-3}{4}$ $\boxed{y = \frac{-3}{4}x + 5}$

Part A: Write an equation for line B.

Part B: Write an equation for line T.

point: (0, 5) $y = mx + b$

$m = \frac{4}{3}$ $\boxed{y = \frac{4}{3}x + 5}$

2. A parallelogram is a four-sided figure whose opposite sides are parallel and equal in length. Alex is drawing parallelogram $ABCD$ on a coordinate plane. The parallelogram has the coordinates $A(4,2)$, $B(0,-2)$, and $D(8,-1)$. $\overline{AB} \parallel \overline{CD}$ and $\overline{AD} \parallel \overline{BC}$

Which of the following coordinates should Alex use for point C?

- ~~A~~ $(6,-3)$
- B $(4,-5)$
- ~~C~~ $(10,-3)$
- ~~D~~ $(4,3)$

$\overline{AB}$ $M = \frac{-2-2}{0-4} = \frac{-4}{-4} = 1$

$\overline{CD}$

A $M = \frac{-3-(-1)}{6-8} = \frac{-2}{-2} = 1$

B $M = \frac{-5-(-1)}{4-8} = \frac{-4}{-4} = 1$

~~C~~ $M = \frac{-3-(-1)}{10-8} = \frac{-2}{2} = -1$

~~D~~ $M = \frac{3-(-1)}{4-8} = \frac{4}{-4} = -1$

$\overline{AD} \parallel \overline{BC}$

$\overline{AD}$ $M = \frac{2-(-1)}{4-8} = \frac{3}{-4}$

A $M = \frac{-3-(-2)}{6-0} = -\frac{1}{6}$

B $M = \frac{-5-(-2)}{4-0} = \frac{-3}{4}$

Introduction to Coordinate Geometry

Coordinate geometry involves placing geometric figures in a coordinate plane.

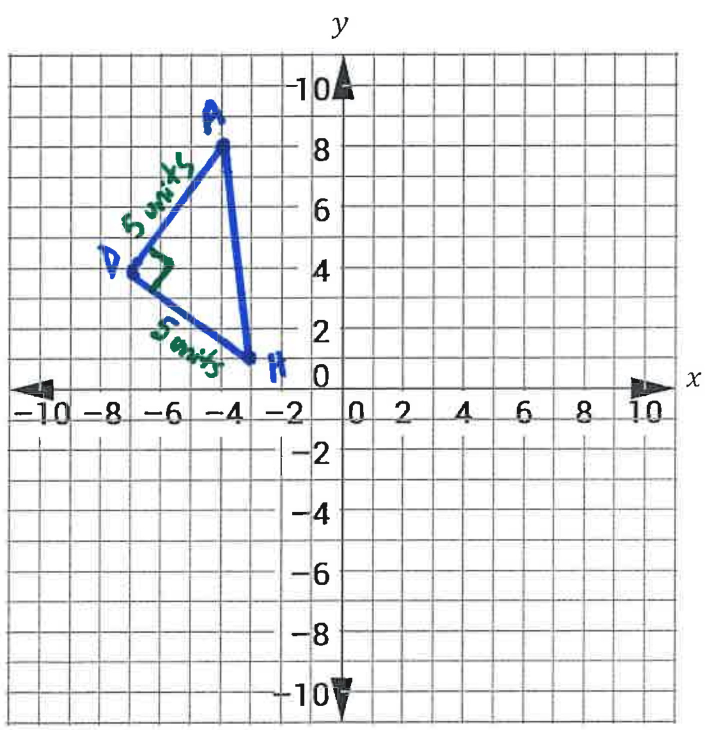
So far in this course, we have used coordinates in the following ways:

	Formula	Description
Midpoint Formula	$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$	halfway point
Distance Formula	$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$	length of a segment
Slope Formula	$\frac{y_2-y_1}{x_2-x_1}$	The rate of change $\Delta Y/\Delta X$

$\frac{\text{vertical change}}{\text{horizontal change}}$

Let's Practice!

1. Given $A(-4,8)$, $D(-7,4)$, and $H(-3,1)$, plot the points, and trace the triangle.



a. What is the perimeter of the triangle? Round to the nearest hundredth. ↳ sum of the lengths of each side of a polygon

$AD: \sqrt{3^2 + 4^2} = \sqrt{9+16} = \sqrt{25} = 5$ units

$DH: \sqrt{4^2 + 3^2} = \sqrt{16+9} = \sqrt{25} = 5$ units

$AH: \sqrt{1^2 + 7^2} = \sqrt{1+49} = \sqrt{50} \approx 7.07$ units

$\text{Perimeter}_{\triangle ADH} = 17.07$ units

b. Prove that $m\angle ADH = 90°$ using the slopes of $\overline{AD}$ and $\overline{DH}$.

$\perp \rightarrow$ slopes are opposite reciprocals

$m_{AD} = \frac{4}{3}$

$m_{DH} = \frac{-3}{4}$

$AD \perp DH$, because $\frac{4}{3}$ is the opposite reciprocal of $\frac{-3}{4}$.

$\therefore m\angle ADH = 90°$

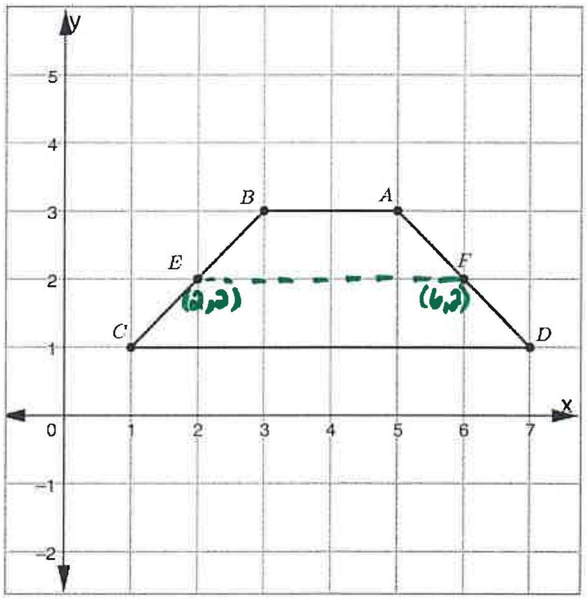
c. Find the area of the triangle. Round to the nearest hundredth.

$A_{\triangle} = \frac{1}{2} b \cdot h$

$A_{\triangle ADH} = \frac{1}{2}(5)(5)$

$A_{\triangle ADH} = 12.5$ square units

2. Consider trapezoid $BADC$ in the figure below.



Given that E is the midpoint of $\overline{CB}$ and F is the midpoint of $\overline{AD}$, show that $\overline{BA} \parallel \overline{EF} \parallel \overline{CD}$ using a paragraph proof.

Given: E is the midpoint of $\overline{CB}$.
F is the midpoint of $\overline{AD}$.
$\mathrm{B}\,(3,3), \mathrm{A}\,(5,3), \mathrm{D}\,(7,1)$, and $C\,(1,1)$

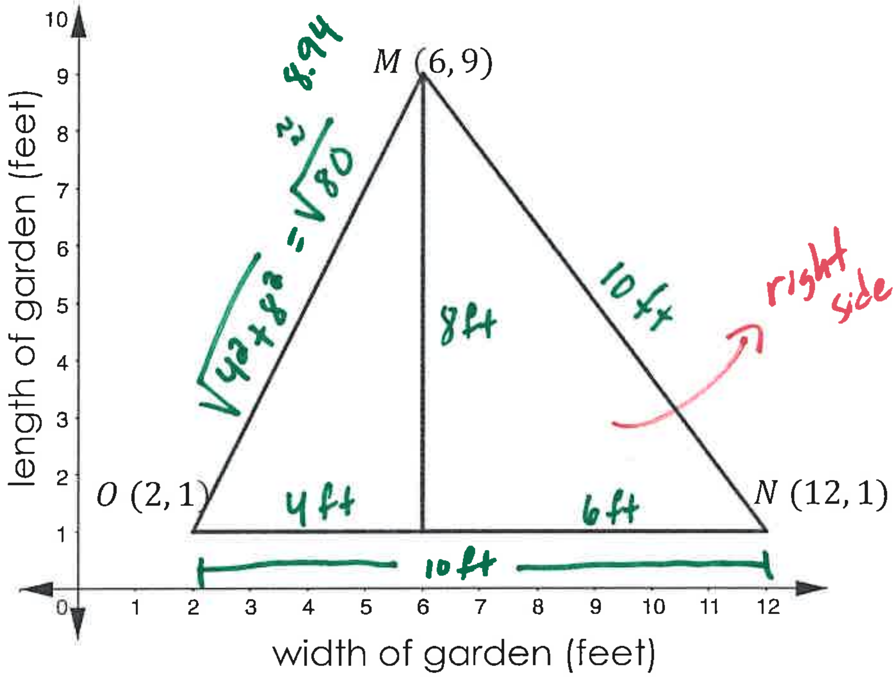
Prove: $\overline{BA} \parallel \overline{EF} \parallel \overline{CD}$

Since E is the midpoint of $\overline{CB}$, E is $\left(\frac{3+1}{2}, \frac{3+1}{2}\right) = (2,2)$.
Since F is the midpoint of $\overline{AD}$, F is $\left(\frac{5+7}{2}, \frac{3+1}{2}\right) = (6,2)$.
Now, the slopes are: $m_{CD} = \frac{0}{6} = 0$; $m_{EF} = \frac{0}{4} = 0$; $m_{BA} = \frac{0}{2} = 0$.

Since they all have the same slope, $\overline{BA} \parallel \overline{EF} \parallel \overline{CD}$.

Try it!

3. Cherise is planting a vegetable garden in the shape of a triangle. She plans to plant tomatoes on the left side and peppers on the right side of the partition that is perpendicular to $\overline{ON}$.



a. If Cherise has 35 feet of fencing, does she have enough to fence in the entire garden and add the partition? Round your answer to the nearest hundredth.

Fencing need is $10 + 10 + 8 + 8.94 = 36.94$

She needs approximately more than 36.94 ft of fencing.

No, she does not have enough fencing.

b. Each pepper plant will need at least 4 square feet of space to produce the most peppers. What is the maximum number of pepper plants she can plant in the right side of her garden?

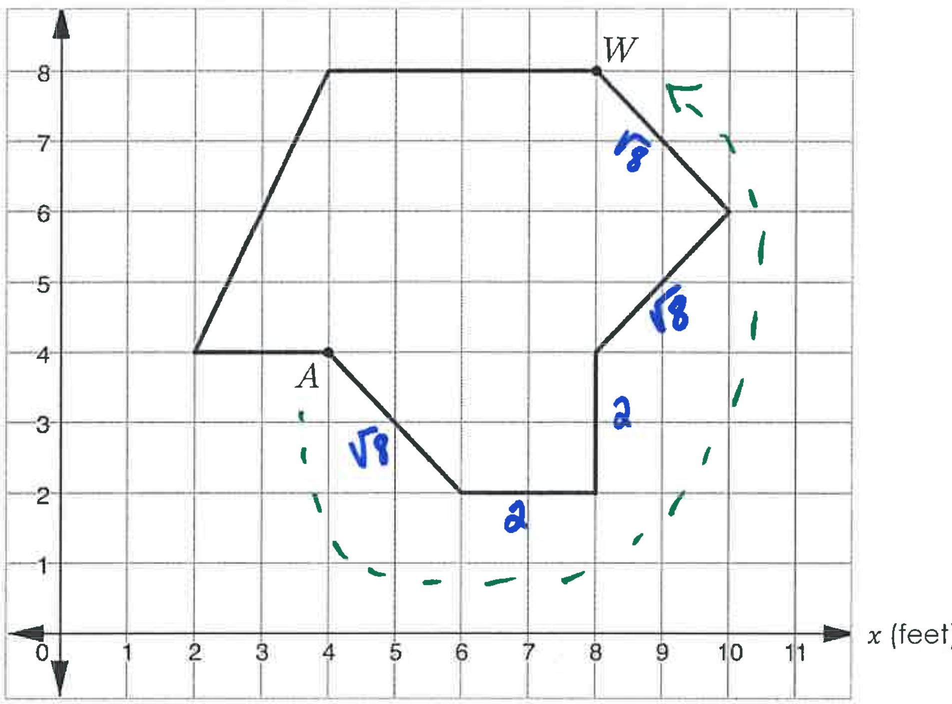
$A_{\text{right side}} = \frac{1}{2}(6)(8) = \frac{1}{2}(48) = 24 \text{ ft}^2$

$24/4 = 6$ pepper plants

BEAT THE TEST!

1. Jerome and Erik start their hike at point A and follow the trail in the counter clockwise direction. They stop at point W to eat lunch.

(6, 5)



How many total miles have Jerome and Erik hiked when they stop for lunch? Round your answer to the nearest tenth of a mile.

$(\sqrt{8} + 2 + 2 + \sqrt{8} + \sqrt{8})$ feet

$(4 + 3\sqrt{8})$ feet

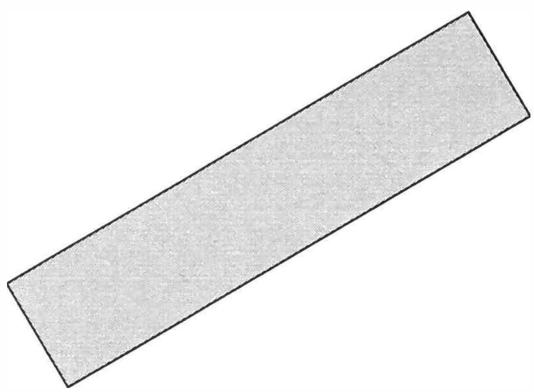
≈ 12.5 feet

Section 1 – Topic 11
Basic Constructions – Part 1

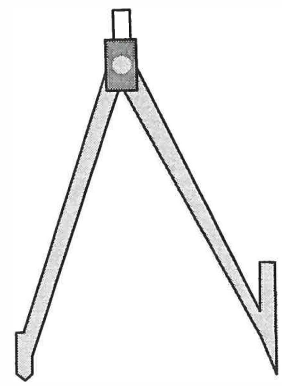
What do you think the term ***geometric constructions*** implies?

Designing with tools

The following tools are used in geometric constructions.



Straightedge



Compass

Which of the tools can help you draw a line segment?

straightedge

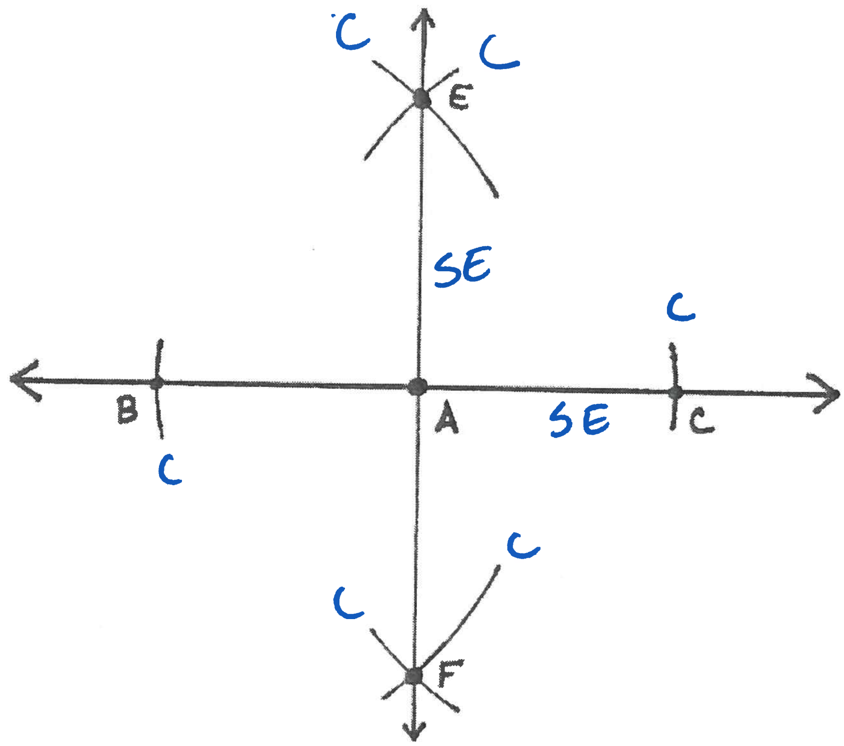
Which of the tools can help you draw a circle?

compass

Constructions also involves labeling points where lines or arcs intersect.

An ***arc*** is a section of the circumference of a circle, or any curve.

Consider the following figure where $\overline{EF}$ was constructed perpendicular to $\overline{BC}$.

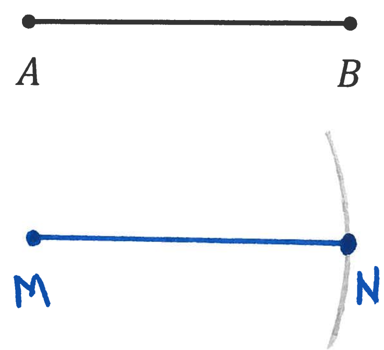


Label each part of the figure that shows evidence of the use of a straightedge with the letters SE.

Label each part of the figure that shows evidence of the use of a compass with the letter C.

Let's Practice!

1. Follow the instructions below for copying $\overline{AB}$.



Step 1. Mark a point M that will be one endpoint of the new line segment.

Step 2. Set the point of the compass on point A of the line segment to be copied.

Step 3. Adjust the width of the compass to point B. The width of the compass is now equal to the length of $\overline{AB}$.

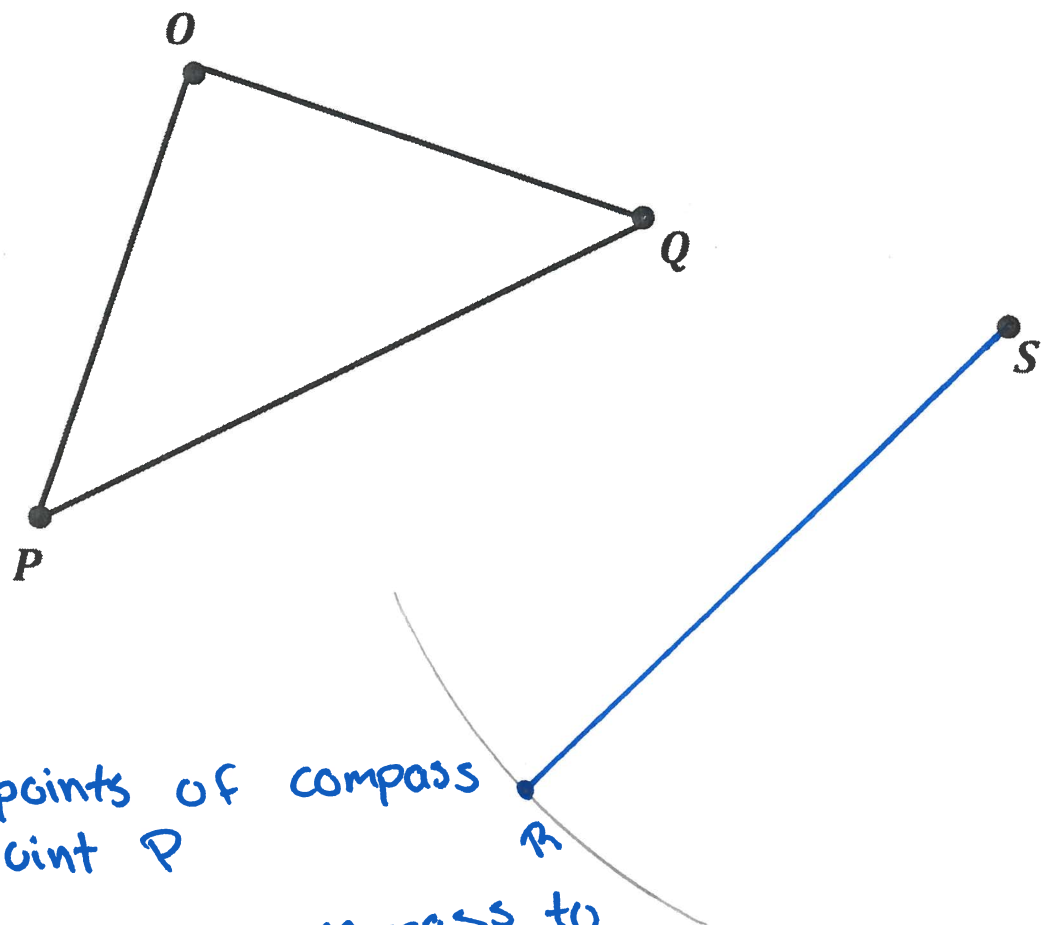
Step 4. Without changing the width of the compass, place the compass point on M. Keeping the same compass width, draw an arc approximately where the other endpoint will be created.

Step 5. Pick a point N on the arc that will be the other endpoint of the new line segment.

Step 6. Use the straightedge to draw a line segment from M to N.

Try It!

2. Construct $\overline{RS}$, a copy of $\overline{PQ}$. Write down the steps you followed for your construction.



① Set points of compass on point P

② Adjust width of compass to be width of $\overline{PQ}$

③ Without changing width of compass, put point of compass on S, draw an arc.

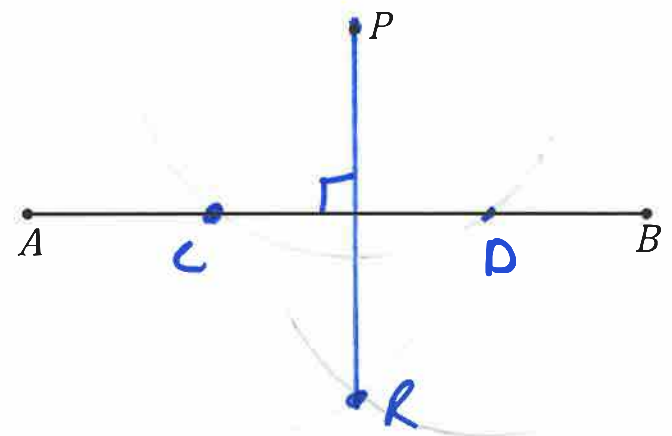
④ Pick a point on arc, draw line segment.

Basic Constructions – Part 2

In the constructions of line segments, we can do more than just copy segments. We can construct lines that are parallel or perpendicular to a given line or line segment.

Let's Practice!

1. Following the steps below, construct a line through P that is perpendicular to the given line segment $\overline{AB}$.



Step 1. Place the point of the compass on point P, and draw an arc that crosses $\overline{AB}$ twice. Label the two points of intersection C and D.

Step 2. Place the compass on point C and make an arc above $\overline{AB}$ that goes through P, and a similar arc below $\overline{AB}$.

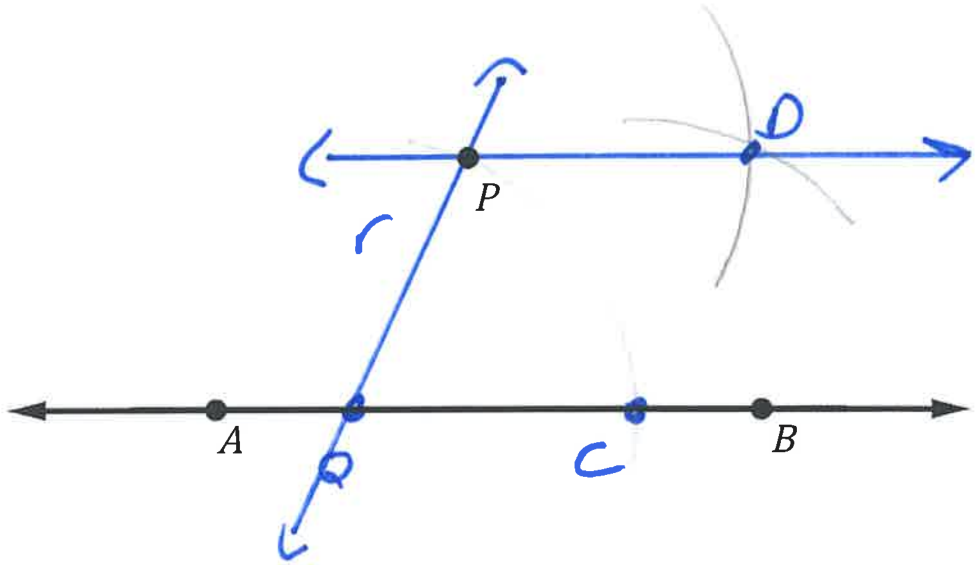
Step 3. Keeping the compass at the same width as in step 2, place the compass on point D, and repeat step 2.

Step 4. Draw a point where the arcs drawn in Step 2 and Step 3 intersect. Label that point R.

Step 5. Draw a line segment through points P and R, making $\overline{PR}$ perpendicular to $\overline{AB}$.

Try It!

2. Following the steps below, construct a line segment through P that is parallel to the given line segment $\overline{AB}$.



Step 1. Draw line r through point P that intersects $\overline{AB}$.

Step 2. Label the intersection of line r and $\overline{AB}$ point Q.

Step 3. Place your compass on point Q, set the width of the compass to point P, and construct an arc that intersects $\overline{AB}$. Label that point of intersection point C.

Step 4. Using the same setting, place the compass on point P, and construct an arc above $\overline{AB}$.

Step 5. Using the same setting, place the compass on point C, and construct an arc above $\overline{AB}$ that intersects the arc drawn in step 4. Label this intersection point D.

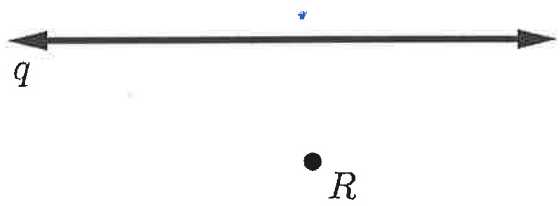
Step 6. Draw $\overline{PD}$.

STUDY EDGE TIP

This construction is for parallel lines using the rhombus method. Later on, we will learn about the properties of rhombi. The construction of parallel lines is also the construction of a rhombus.

BEAT THE TEST!

1. Consider the figure below.



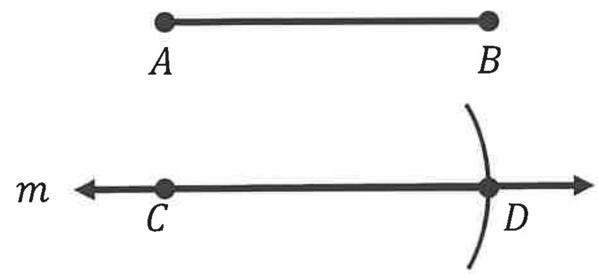
Celine attempted to construct a line through point R that is perpendicular to line q. In her first step, she placed the point of the compass on point R, and drew an arc that crossed line q twice. She labeled the two points of intersection A and B. Then, Celine placed the compass on point A and made an arc above line q that went through R; repeating the same process from point B. Finally, she drew a line from R crossing line q.

Part A: Celine's teacher pointed out that the construction is missing a very crucial step. Determine what the missing step is and why it is so crucial for this construction. Draw arcs above q so they intersect. We can then have intersection point ~~B~~. to connect R.

Part B: Another student in the classroom, Lori, suggested that Celine can construct a line parallel to q through R by drawing a horizontal line. The teacher also pointed out that Lori's claim was incorrect. Explain why.

There is no way to know it is parallel w/o constructing it.

2. Consider the diagram shown below.

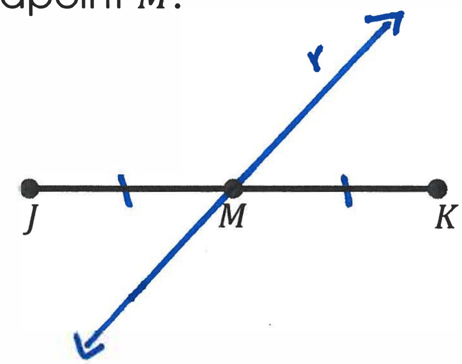


Which of the following statements best describes the construction in the diagram?

Ⓐ $\overline{AB} \parallel \overline{CD}$.
Ⓑ $\overline{AB} \cong \overline{CD}$.
Ⓒ C is the midpoint of m.
Ⓓ D is the midpoint of m.

Section 1 – Topic 13
Constructing Perpendicular Bisectors

Consider $\overline{JK}$ with midpoint M.



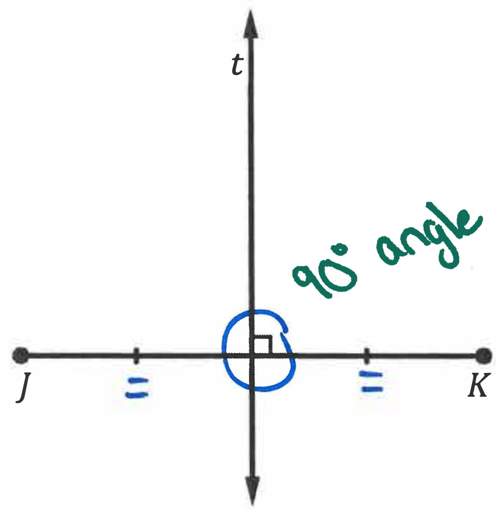
Draw a line through point M and label it r.

Line r is the segment bisector of $\overline{JK}$.

A ***bisector*** divides lines, angles, and shapes into two equal parts.

A ***segment bisector*** is a line, segment, or ray that passes through another segment and cuts it into two congruent parts.

Consider $\overline{JK}$ and line t.



Line t is the ***perpendicular bisector*** of $\overline{JK}$. Make a conjecture as to why line t is called the perpendicular bisector of $\overline{JK}$.

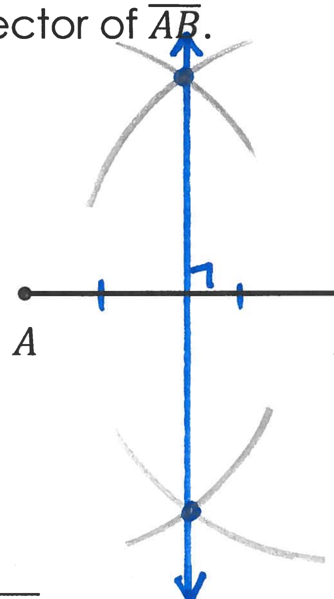
The 90° angle mark tells me that line t is perpendicular to $\overline{JK}$ and the tick marks tell me that t is also bisecting $\overline{JK}$.

STUDY EDGE TIP

When you make a ***conjecture***, you make an educated guess based on what you know or observe.

Let's Practice!

1. Follow the instructions below for constructing the perpendicular bisector of $\overline{AB}$.



Step 1. Start with $\overline{AB}$.

Step 2. Place your compass point on A, and stretch the compass more than halfway to point B.

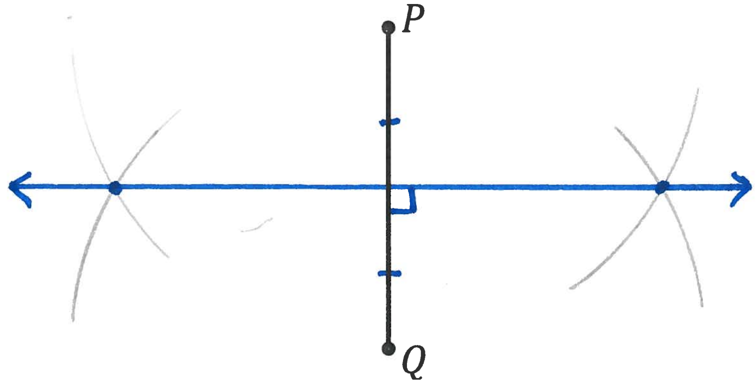
Step 3. Draw large arcs both above and below the midpoint of $\overline{AB}$.

Step 4. Without changing the width of the compass, place the compass point on B. Draw two arcs so that they ***intersect*** the arcs you drew in step 3.

Step 5. With your straightedge, connect the two points of where the arcs intersect.

Try It!

2. Consider $\overline{PQ}$.



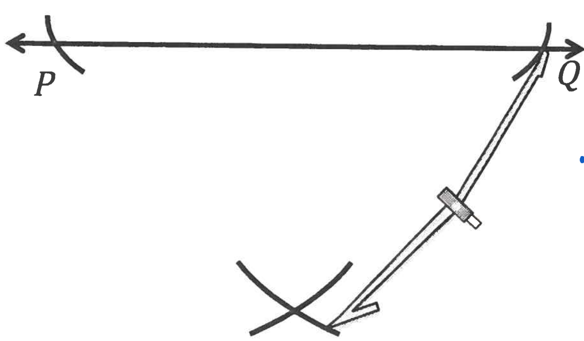
a. Construct the perpendicular bisector of $\overline{PQ}$ shown above.

b. Consider $\overline{AB}$, which is parallel to $\overline{PQ}$. Is the perpendicular bisector of $\overline{PQ}$ also the perpendicular bisector of $\overline{AB}$? Justify your answer. Not necessarily!
We do not know the location of $\overline{AB}$.
In order for the perpendicular bisector of $\overline{PQ}$ to be the perpendicular bisector of $\overline{AB}$, $\overline{AB}$ needs to be the product or outcome of a horizontal shift from $\overline{PQ}$

3. Consider the diagram below. What do you need to check to validate the construction of a perpendicular bisector?

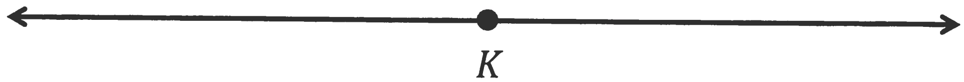
• We need another pair of interceting areas from P to R and from Q to R.

• We also need to make sure that the compass width from P to R and from Q to R is the same width used to draw the two arcs below $\overleftrightarrow{PQ}$.

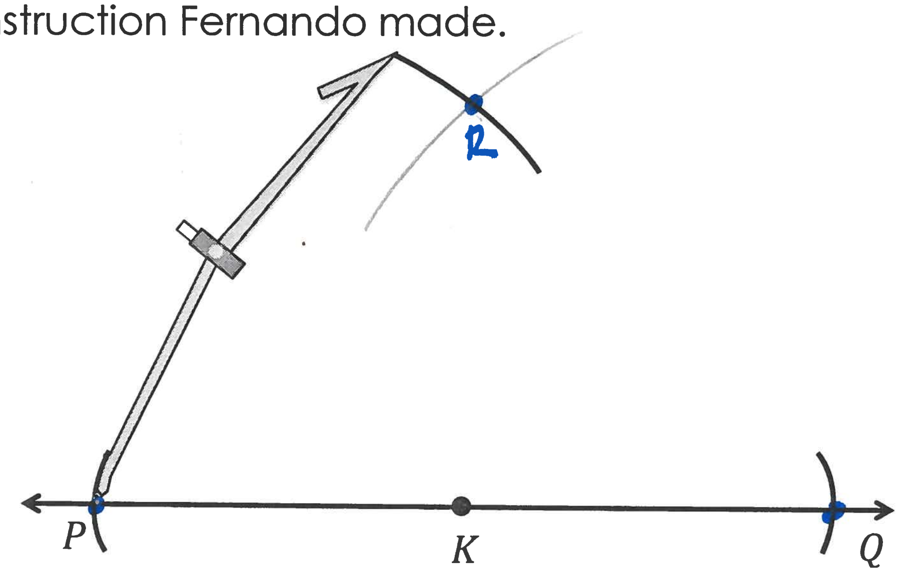


BEAT THE TEST!

1. Fernando was constructing a perpendicular line at a point K on the line below.



The figure below represents a depiction of the partial construction Fernando made.

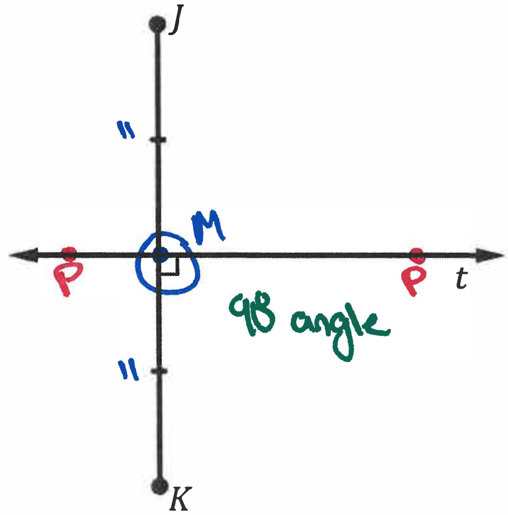


What should the next step be?

- ~~A~~ Increase the compass to almost double the width to create another line.
- ~~B~~ From P, draw a line that crosses the arc above K.
- C Without changing the width of the compass, repeat the drawing process from point Q, making the two arcs cross each other at a new point called R.
- ~~D~~ Close the compass and use the straight edge to draw a line from the midpoint of the arc to point K.

Section 1 – Topic 14
Proving the Perpendicular Bisector Theorem Using Constructions

Consider $\overline{JK}$ and line t again.



What is the intersection between line t and $\overline{JK}$ called?

midpoint of $\overline{JK}$

Let's Practice!

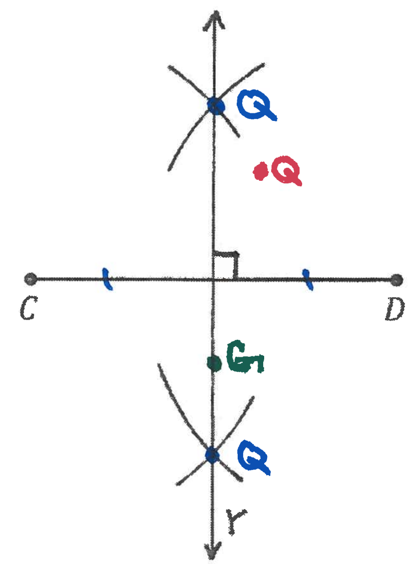
1. Using the above diagram where line t is the perpendicular bisector of $\overline{JK}$, let M be the point where line t and $\overline{JK}$ intersect, and let P be any point on line t.

   a. Suppose that P lies on $\overline{JK}$. What conclusions can you draw about the relationship between $\overline{JP}$ and $\overline{KP}$? Explain.
   On poin M
   $\overline{JP} \cong \overline{KP}$ because P will be located on M, which is the midpoint of $\overline{JK}$.

   b. Suppose that P does not lie on $\overline{JK}$. What conclusions can you draw now about the relationship between $\overline{JP}$ and $\overline{KP}$? Explain. Still, $\overline{JP} \cong \overline{KP}$ because t is perpendicular to $\overline{JK}$, so any location of P on t will preserve congruence between $\overline{JP}$ and $\overline{KP}$. Compass confirms it.

Try It!

1. Suppose that C and D are two distinct points in the plane and a student drew line r to be the perpendicular bisector of $\overline{CD}$ as shown in the diagram below.



a. If G is a point on r, show that G is equidistant from C and D. The width of the compass is exactly the same from D to G and from C to G.

b. Conversely, use a counterexample to show that if Q is a point which is equidistant from C and D, then Q is a point on r. We can adjust the compass and see that the same width used from C to Q will watch the width from D to Q. If we put Q anywhere but on r, that won't work.

c. Determine if the following statement is true.

The perpendicular bisector of $\overline{CD}$ is exactly the set of points which are equidistant from C and D.

True!!!

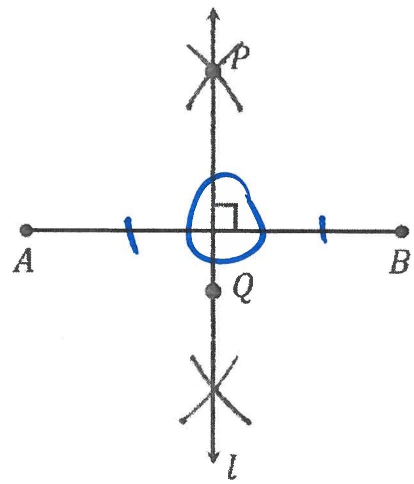
TAKE NOTE! *Postulates & Theorems*

Perpendicular Bisector Theorem

If a point is on the perpendicular bisector of a segment, then it is equidistant from the endpoints of the segment. The **converse** of this theorem is also true.

BEAT THE TEST!

1. Consider the following diagram, A and B are two distinct points in the plane and line l is the perpendicular bisector of $\overline{AB}$.



Yozef and Teresa were debating whether P and Q are both on l. Circle the correct response. Justify your answer.

Yozef's work	Teresa's work
I measured the distance between A and P, and B and P, and the width of the compass was the same for both. Same happened between A and Q, and B and Q. Therefore, P is equidistant from A and B, and Q is equidistant from A and B. P and Q are both on line l justified by ~~of~~ the Converse of the Perpendicular Bisector Theorem.	~~P is on the intersection of the arcs drawn in the construction process above segment AB, so the width of the compass is the same from A to P and from B to P. However, Q is not on the intersection of the arcs drawn below the segment, so it is not equidistant from A and B. In conclusion, P is on the perpendicular bisector l but Q is not on it.~~

need to use distance to justify it.

Great job! You have reached the end of this section. Now it's time to try the "Test Yourself! Practice Tool," where you can practice all the skills and concepts you learned in this section. Log in to Math Nation and try out the "Test Yourself! Practice Tool" so you can see how well you know these topics!

Test Yourself! Practice Tool